# SIMULTANEOUS ORBIT FITTING OF STELLAR STREAMS: CONSTRAINING THE GALACTIC DARK MATTER HALO

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# ABSTRACT

The Milky Way Galaxy serves as a laboratory for testing models of galaxy formation. Discovering the nature of dark matter is often cited as the second most important problem in astrophysics, preceded only by dark energy. Mapping the structure and dynamics of the Milky Way Galaxy can tell us how galaxies form, and place constraints on the properties of dark matter.

We can map the distribution of dark matter in the Milky Way using tidal streams, collections of stars that have been gravitationally stripped from satellite dwarf galaxies and globular clusters. By knowing the positions and velocities of these stars, and assuming they came from a compact source, we can follow them back in time and constrain the shape of the Milky Way dark matter halo.

This Thesis presents a method that allows us to constrain the parameters of a static Galactic gravitational potential using the data from any number of tidal debris streams. The method is tested on simulated tidal streams, and successfully recovers the original model parameters in most cases. The importance of simultaneously fitting the measured rotation curve of the Milky Way is explored, and the strengths and weaknesses of the algorithm are discussed.

The orbit fitting algorithm is applied independently to the Stream of Grillmair and Dionatos (GD-1), the Orphan Stream, and the Cetus Polar Stream (CPS). We show that no known globular cluster or dwarf galaxy in the Milky Way has kinematics consistent with being the progenitor of the GD-1 stream. The Orphan Stream constrains the Milky Way dark matter halo as having a mass at the low end of previous measurements, giving a best fit halo speed of  $v_{halo} = 73 \pm 24$  km s<sup>-1</sup>, compared to typical values of  $v_{halo} \approx 115$  km s<sup>-1</sup>. A lower halo speed implies a less massive halo.

The GD-1 and Orphan streams are then fit simultaneously with the Sagittarius Dwarf Tidal Stream (Sgr), within a triaxial dark matter halo. Results for restricted triaxial cases are shown to be consistent with previous authors. Simultaneous fits within an unrestricted triaxial halo (free to rotate in any direction) give flattenings  $q_x = 1.33 \pm 0.16$ ,  $q_z = 1.52 \pm 0.14$  and XYZ pitch-roll-yaw Euler orientation angles of  $(\theta, \phi, \psi) = (-50^{\circ} \pm 18^{\circ}, 86^{\circ} \pm 11^{\circ}, 1^{\circ} \pm 6^{\circ})$ . The best fit halo speed and scale length are  $v_{halo,t} = 126 \pm 9$  km s<sup>-1</sup> and  $d_{halo,t} = 22.2 \pm 3.3$  kpc, respectively. The  $\phi$ Euler angle is broadly consistent with that found for the stellar halo by Newberg et al. (2006). The significance of these orientation angles within the context of Galaxy formation and evolution are discussed.

Utilizing the orbit fits to the Orphan Stream, a novel technique is presented to fit the density of F-turnoff stars along the stream utilizing semi-analytic N-body simulations. Baseline estimates of the mass and scale radius of the Orphan Stream progenitor are obtained. We discuss the development of a stellar density fitting algorithm, which is implemented on the Milkyway@home volunteer computing platform.

# CHAPTER 1 INTRODUCTION

## 1.1 Historical Overview

From the time when mankind first gazed into the night sky and saw the Milky Way, we have sought to understand its nature, structure, and history. Originally understood in Chinese culture as the "Silver River," Uralic cultures as the "Bird's Path," and Japanese culture as the "River of Heaven," views during antiquity shifted from predominantely mythological to more physical in nature. The Ancient Greeks first proposed the Milky Way as being composed of stars, and by a millennium and a half later, Galileo, Kant, and Herschel had formulated the view of the Milky Way as a large rotating body of stars similar to our Solar System. It wasn't until Curtis (1917), through observations of supernovae in "nebulae," deduced that they were much farther from the Sun than originally thought. This transformed "spiral nebulae" such as M31 into large galaxies, and raised the possibility that the Sun itself was within a similar structure. This conclusion was reaffirmed by Hubble (1925), via observations of Cepheid variables in M31.

The advent of infrared and radio surveys have proved decisive in the understanding of Milky Way structure. When looking at the Galactic center via optical telescope, one's view is obstructed by dust and gas. Infrared surveys are unaffected by this, and reveal a spherical collection of stars at the center of the Galaxy termed the Galactic bulge. Within this bulge is a central concentration called the nucleus. Doppler shift observations of nucleus stars reveal velocities consistent with orbits around a point mass of  $4 \times 10^6 M_{Sun}$ , suggesting a supermassive black hole at the Galactic center. Additionally, radio surveys reveal stellar overdensities occuring at regular intervals from the Galactic center. These are understood as spiral arms, providing evidence that the Milky Way is a spiral galaxy (Matsumoto et al. 1982; Georgelin, et al. 1976).

While the view of the Milky Way itself was being refined, observations were also being taken that showed it is not alone in its neighborhood. The Magellanic Clouds, known since antiquity, were now coming into view as being dwarf galaxies orbiting the Milky Way. Discoveries of IC10 in 1887, 1 Zwicky 18 in the 1930s, the Aquarius Dwarf in 1959 led to the view that the Milky Way is host to orbiting dwarf galaxies.

In the midst of discoveries of Milky Way neighbors, the picture of the Milky Way itself was still a matter of contention. In the mid-20th century, the paradigm consisted of a spiral disk and central bulge with the Sun being roughly 8 kiloparsecs from the Galactic center. The exact value is still controversial. As telescope technology advanced, a new population of stars above and below the disk was revealed. At first, a population on the order of 2 kpc above and below the luminous disk was identified, and termed the "thick disk" (Gilmore & Wyse, 1985). Following this, observations further out from the Galactic disk revealed another population. These stars are generally old and unenriched with metallic elements, and are termed the Galactic stellar halo. It was postulated that the density of stars in the stellar halo could be fit using a simple power law (e.g. Hawkins, 1984) such as  $\rho \propto r^{-\alpha}$ . This effort did not prove fruitful as examinations of different areas in the sky led to different fits for the exponent  $\alpha$ . Sophisticated models such as the Bahcall-Soniera (1980) model were developed, which utilized observations within the solar neighborhood, and a global distribution of matter consisting of an exponential disk and de Vaucouleurs spheroid. While these models are immensely successful at predicting various Galactic properties, their assumption of a smooth background precludes any discussion of Galactic substructure.

This dilemma remained until the development of large scale, multi-color, highly calibrated, all sky photometric surveys such as the Two-Micron All Sky Survey and the Sloan Digital Sky Survey, which allowed for large areas of the sky to be viewed simultaneously. In Figure 1.1 from Newberg et al. (2002), we see the first detection of large scale substructure in the Milky Way Halo. The overdensities at  $(RA, g_0) = (30^\circ, 21)$  and  $(RA, g_0) = (210^\circ, 22)$ , which cannot be explained as defects in a smooth background, are the disrupted remnants of the Sagittarius Dwarf Galaxy (Ibata et al., 1994).

This discovery shed light on the failures of the previous methods used to fit



Figure 1.1 Histogram in  $g_0$  vs. RA of turnoff stellar density on the celestial equator from Newberg, et al. (2002). The overdensity at  $(RA, g_0) = (30^\circ, 21)$  is the first detection of the Sagittarius Dwarf (Sgr) tidal tail in F-turnoff stars. The overdensity between RA = 240° and 330° is identified as the Galactic stellar halo; the Sgr overdensity cannot be a product of a smooth background component. The overdensity at  $(RA, g_0) = (210^\circ, 22)$  is also a Sgr tidal tail, on the other side of the Galaxy. The linear overdensity at RA = 229° is the globular cluster Palomar 5. The radial length of this overdensity illustrates the variations of intrinsic brightnesses in F-turnoff stars, since these stars are all at the same distance from the Sun. Figure reproduced with permission.

the density power law exponent. An examination of a small sky area could be contaminated by halo substructure, and only by examining the entire sky can we see whether there is substructure in a particular region. However, all is not lost with regards to fitting the density power law exponent. One would simply have to remove all pieces of substructure from the Galaxy, and fit the remaining population to determine the density profile . Cole, et al. (2008) developed a novel technique to separate tidal debris from SDSS stripes, assuming a particular form for the stellar spheroid density. This technique was used to map the Sagittarius stream around the entire SDSS footprint. Future work in this field will involve simultaneously fitting a spheroid background density and removing stream components to determine the best fit density power law exponent.

Once substructure was identified in the stellar halo, a flood gate was opened that permitted the discovery of new halo substructures. Grillmair et al. (1995); Leon, Meylan, & Combes (2000); and Testa et al. (2000) searched for tidal streams of globular clusters using photographic data. Odenkirchen et al. (2001) identified tidal tails around the Palomar 5 globular cluster, which were further expanded to 22° by Grillmair and Dionatos (2006). Rockosi et al. (2002) developed a matched filter technique that is able to select stellar populations out of large sky surveys such as SDSS. This technique was used by Grillmair and Dionatos (2006) to discover a 63° globular cluster stream (GD-1) and Grillmair (2009) to isolate four previously unknown globular cluster tidal streams from the SDSS footprint. Belokurov et al. (2006) and Grillmair & Johnson (2006) isolated tidal tails extending from the NGC 5466 globular cluster. Constraints on the Anti-center stream were established by Grillmair, Carlin and Majewski (2008). The Orphan Stream was independently discovered by Grillmair (2006) and Belokurov et al. (2006), and was elaborated upon by Newberg, Willett, Yanny, and Xu (2010). New tidal streams are continuing to be discovered in the SDSS footprint via the matched filtering technique (Grillmair, private conversation), however not all streams are discoverable as visible overdensities in photometric data. The Cetus Polar Stream, discovered by Newberg, Yanny and Willett (2009) was isolated due to its characteristic velocities. Similarly, a moving group 50 kpc from the Sun was discovered by Harrigan et al. (2010). Whether discovered spatially or using velocities, the abundance of data provided by the SDSS and the availability of advanced data analysis techniques have made it apparant that the Galaxy is host to many pieces of substructure.

Even after the discovery of the stellar halo and its substructure, mysteries remained in large-scale Galactic structure. The rotation speeds of stars around the center of the Milky Way are consistent with being a constant value of v = 220 km s<sup>-1</sup> out to distances of 60 kpc from the Galactic center. This suggests that there is more mass in the Milky Way than can be accounted for in stars. This extra mass is in the form of a dark matter halo, which extends farther from the Galactic center than the stellar halo.

Understanding the structure of the dark matter halo is an active research topic. In this work, we will endeavour to fit orbits to tidal streams in the Galactic environment. The remainder of this chapter will discuss basic astronomy concepts such as coordinate systems, various Galactic models that will be used throughout this Thesis, and finally will describe the process of tidal disruption. In Chapter 2 a general orbit fitting method will be formulated. Chapter 3 will describe a test of the orbit fitting method on simulated tidal streams, and in Chapter 4 fits to actual Galactic tidal streams will be presented. The Thesis will conclude with discussion and future work in Chapter 5.

# **1.2** Galactic Coordinate Systems

Before we can begin to understand orbits in the Galactic environment, a set of coordinate systems needs to be devised. Due to the fact that observations are conducted in a Earth-centered frame, and orbits are modeled in a Galaxy-centered frame, a clear definition of these systems must be established before proceeding.

Stars in the night sky are observed utilizing the Equatorial coordinate system, which is an Earth-based system composed of two angles: right ascension (reported as RA or  $\alpha$ ), and declination (reported as Dec or  $\delta$ ). Right ascension is the angle of an object east of the Sun at the March equinox. Declination is the angle of an object from the celestial equator (e.g. Duffett-Smith, 1988). Due to precession of the Earth over time, the equatorial coordinates need to be defined with reference to a specific date and time. Unless otherwise noted, all equatorial coordinates in this Thesis are of the J2000 system.

The Galactic Cartesian coordinate system used in this Thesis is defined as a three-dimensional, right-handed coordinate system. The origin is the center of the Galaxy. The X axis points from the Galactic center away from the Sun, the Y axis points in the direction of Galactic rotation, and the Z axis points perpendicular to the X-Y plane in a right-handed fashion. In this system, the Sun is located at  $(X_{GC}, Y_{GC}, Z_{GC}) = (-R_{Sun}, 0, 0).$ 

Another commonly used coordinate system is the Sun-centered Cartesian coordinate system. The directions of the unit vectors in this system is the same as in the Galactic system. The only difference is the center of the coordinate system is shifted along the X axis to the Sun. The center of the Galaxy is then located at  $(X_{SC}, Y_{SC}, Z_{SC}) = (R_{Sun}, 0, 0).$ 

The Galactic coordinate system (not to be confused with the Cartesian one defined above) is an Earth-based system also composed of two angles: Galactic longitude (reported as l) and Galactic latitude (reported as b). The center of the coordinate system,  $(l, b) = (0^{\circ}, 0^{\circ})$ , points toward the center of the Galaxy (along the X axis defined above). l increases in a counterclockwise fashion.  $b = +90^{\circ}$  points toward the north Galactic pole, while  $b = -90^{\circ}$  points toward the south Galactic pole.

Two additional specialized coordinate systems need to be specified. The first is the Orphan Stream coordinate system of Newberg, et al. (2010). This system was developed to easily model the Orphan Stream, and consists of a spherical system where the Orphan Stream lies along its equator. This system is composed of two angles:  $\Lambda_{Orphan}$ ,  $B_{Orphan}$ . The center of the coordinate system,  $(\Lambda_{Orphan}, B_{Orphan}) =$  $(0^{\circ}, 0^{\circ})$  lies along the Orphan stream, at the intersection of that stream and the leading tail of the Sagittarius Dwarf Tidal Stream.  $\Lambda_{Orphan}$ ,  $B_{Orphan}$ ) is accomplished via the following transformation:

$$\begin{array}{c} \cos B_{\mathrm{Orphan}} \cos \Lambda_{\mathrm{Orphan}} \\ \cos B_{\mathrm{Orphan}} \sin \Lambda_{\mathrm{Orphan}} \\ \sin B_{\mathrm{Orphan}} \end{array} \end{array} \right) = \mathcal{M} \left( \begin{array}{c} \cos b \cos l \\ \cos b \sin l \\ \sin b \end{array} \right),$$

where

$$\mathcal{M} = \begin{pmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{pmatrix}$$

where the angles  $(\phi, \theta, \psi)$  were determined by Newberg, et al. (2010) to be (128.79°, 54.39°, 90.70°).

The other is the Galaxy-centered, Sagittarius Dwarf (Sgr) Tidal Stream system of Majewski, et al. (2003), which is a similar idea to the previous system but for a different stream. This system is composed of two angles:  $\Lambda_{Sgr,GC}$  and  $B_{Sgr,GC}$ . The center of the coordinate system,  $(\Lambda_{Sgr,GC}, B_{Sgr,GC}) = (0^{\circ}, 0^{\circ})$ , is located at  $(l, b) = (5.6^{\circ}, -14.2^{\circ})$ , the Sagittarius Dwarf Galaxy core.  $\Lambda_{Sgr,GC}$  increases in the direction of the leading tidal tail (roughly along the Z-axis). The coordinate system is defined by the angles  $(\phi, \theta, \psi) = (183.8^{\circ}, 76.5^{\circ}, 201.6^{\circ})$ , with a rotation center of  $(X_{GC}, Y_{GC}, Z_{GC}) = (-8.51, -0.21, -0.05)$  kpc.

For the purpose of this Thesis, stars are considered to be point particles. The orbit of a star is fully determined by its position and velocity in the Galactic Cartesian coordinate system, as well as the gravitational field of the Galaxy. While, in reality, the Galaxy is composed of an aggregation of many distinct and lumpy components (Newberg, et al. 2002), for our purposes it will be modeled as a superposition of smooth gravitational potentials. The specific form of these potentials is described in a later subsection.

Stars within a smooth Galactic gravitational potential evolve according to Newton's Laws. No relativistic effects are considered throughout the course of this work. The fact that, in general, the Galactic gravitational potential is not spherically symmetric results in an orbit of a tidal stream that cannot simply be modeled as a great circle on the sky. Even if the potential were perfectly spherical, great circle modeling is insufficient because the point of observation (the Sun) is not located at the Galactic center.

## **1.3** Galactic Potentials

In the completely general case, the gravitational field of the Galaxy can be understood as the superposition of potentials from each of the individual mass particles that make it up. Stars can be simply modeled as spherical gravitational potentials. Gas and dark matter in the Galaxy pose difficult modeling challenges. If one could know the positions of all gas particles and dark matter entities, finding the total potential would be trivial. However, gas tends to occur in clumps, and usually only in the newer, more metal rich population in the Galactic disk. Dark matter is believed to exist in large quantities above and below the Galactic disk, in what is termed the dark matter halo. As dark matter particles have yet to be detected, creating a total gravitational field through superposition is not feasible.

To construct a gravitational model of the Galaxy, we must rely on gravitational potentials that have appropriate mass density profiles, and finding parameters such as the total mass and scale length that make the potential appropriate to the Galactic problem. For the purpose of this Thesis, the Galaxy is composed of three parts: a spherical bulge of matter near the center of the Galaxy, a disk potential spanning the diameter of the Galactic disk, and a dark matter halo that is several Galactic diameters wide. These models are not new to this work, and will simply be enumerated below.

#### **1.3.1** System of Units and Relevant Constants

Before discussing Galactic and dwarf potential models, it is necessary to outline the system of units that will be used for this Thesis. As a matter of tradition, the gravitational constant is identically 1. The length unit used is a kiloparsec (kpc,  $10^3$  parsecs (pc) =  $3.09 \times 10^{19}$  m). The Galactic disk is on the order of 30 kpc in diameter. The unit of time is the gigayear (Gyr,  $10^9$  years =  $3.16 \times 10^{16}$ sec). The unit of velocity is the kiloparsec per gigayear (kpc/Gyr = 1.02 km/s). To satisfy the requirement of a unity gravitational constant, the unit of mass is  $M_u = 222288.47 M_{Sun} = 4.45 \times 10^{35} \text{ kg}$ .

Typically, observational velocities for stars in the Galactic environment are reported in kilometers per second. It is a fortunate happenstance that the simulation unit of kpc/Gyr is nearly identical to this. Orbits and simulations will be performed in simulation units, and reported in observational units.

In this Thesis, we adopt a Sun-Galactic center distance of  $R_{Sun} = 8.0$  kpc.

## 1.3.2 Spherical Bulge Model

The gravitational potential of Galactic bulge is a simple spherical potential with a mass  $M_{bulge}$  and scale radius  $r_c$ . The functional form of this potential is given in Equation 1.1, with  $r = \sqrt{X_{GC}^2 + Y_{GC}^2 + Z_{GC}^2}$ . For the remainder of this Thesis, the bulge mass is fixed at  $M_{bulge} = 3.4 \times 10^{10} M_{Sun}$  and the scale radius is fixed at  $r_c = 0.7$  kpc. These values are adopted from Law, et al. (2005).

$$\Phi_{bulge} = \frac{-M_{bulge}}{r+r_c} \tag{1.1}$$

### 1.3.3 Miyamoto-Nagai Disk

Miyamoto and Nagai (1975) (M-N) derived a generalized, flattened, cylindrically symmetric potential that can be used to represent the disk of the Galaxy. The functional form of the gravitational potential is given in Equation 1.2,

$$\Phi_{disk} = \frac{-M_{disk}}{\sqrt{X_{GC}^2 + Y_{GC}^2 + \left(a + \sqrt{Z_{GC}^2 + b^2}\right)^2}}$$
(1.2)

where a and b are the disk scale length and scale height, respectively.  $M_{disk}$  is the total mass of the entire disk potential. Unless otherwise stated, the M-N disk is the default disk model of this Thesis. The parameters of this model are those from Law et al. (2005): a = 6.5 kpc, b = 0.26 kpc, and  $M_{disk} = 1 \times 10^{11} M_{Sun}$ .

### 1.3.4 Exponential Disk

The second and final disk model that will be considered is the Exponential disk. As described in Xue, et al. (2008), this disk potential has the functional form

$$\Phi_{disk} = -\frac{M_{disk} \left(1 - e^{-r/b}\right)}{r} \tag{1.3}$$

where  $r = \sqrt{X_{GC}^2 + Y_{GC}^2 + Z_{GC}^2}$ , b is the disk scale length and  $M_{disk}$  is the total mass of the disk potential. Typical values for the disk scale length are  $b \approx 4$  kpc.

#### 1.3.5 The Galactic Rotation Curve and the Need for Dark Matter

The Galactic rotation curve is the rotation speed of stars in the X-Y plane as a function of distance from the Galactic center. To understand the nature of rotation curves and how they apply to the Galaxy, consider the following two examples:

- Hard Sphere: For the case of a hard sphere rotating with angular momentum L, the rotation speed at radius r, v(r) is linearly proportional to the radius:  $v(r) = \omega r$ .
- Solar System Planets: In the solar system, the planets closer to the Sun orbit with faster speeds than the planets farther away. The rotation speed as a function of radius is inversely proportional to the radius:  $v(r) \propto 1/r$ .

A sample disk+bulge rotation curve is shown in Figure 1.2. The rotation curve for the bulge behaves as a hard sphere, and so see that the blue curve is linear until  $R \approx 0.5$  kpc. The Galactic disk (shown in green) contains stars that orbit the Galactic center in a fashion similar to planets orbiting the Sun, so the disk should behave like the Solar System. The expected, combined total rotation curve is shown in red.

Quite unexpectedly, the rotation curve for the Galaxy does not exhibit this behavior. The velocity increases with radius close to the Galactic center, and shows a slight drop for intermediate radii (1 kpc < r < 5 kpc), but remains at a constant speed  $v_{rot}$  for larger radii, as can be see by the Xue et al. (2008) rotation curve data shown in Figure 1.2. This discovery was made by Vera Rubin in 1970 with Andromeda, but was subsequently shown for our Galaxy.

The accepted explanation for this rotation curve behavior is the presence of additional, unseen matter in the Galaxy that exists predominantly in the region above and below the Galactic disk. This additional mass is taken to be non-baryonic



Figure 1.2 The expected rotation curve for a galaxy only containing a bulge ( $M_{bulge} = 3.4 \times 10^{10} M_{Sun}$ ) and disk ( $M_{disk} = 1.0 \times 10^{11} M_{Sun}$ ) is shown in red. The individual bulge and disk components are shown in blue and green, respectively. The linear behavior for R < 0.5 kpc due to the bulge is apparent, as is the decline in speed as a function of distance for the disk. Shown also is the derived Galactic rotation curve from Simulation 1 of Xue et al. (2008). We see that a purely disk+bulge potential becomes inconsistent with observations for R > 30 kpc.

dark matter. While the true nature of dark matter is well beyond the scope of this Thesis, the gravitational influence of dark matter can nonetheless be modeled for our orbit-fitting purposes. Below we enumerate three well established dark matter gravitational potentials: the logarithmic, triaxial, and NFW dark matter halos.

#### 1.3.6 Logarithmic Halo

Quite simply, to model the Galactic dark matter, the only requirement is to create a potential that has a flat rotation curve for large radii. Consider a particle of constant mass orbiting the Galaxy at radius r with speed v and centripedal acceleration  $a_c$ . The centripedal acceleration of the particle is  $a_c = v/r$ . Since the particle is orbiting solely due to the gravitational field of the Galaxy, the only acceleration on the particle is centripedal. Therefore, the net force on the particle is  $F_{net} = ma_c = mv/r$ . This gives a rotation speed  $v = F_{net}r/m$ . For the speed v to remain constant for all values of r, the net force must be inversely proportional to r:  $F_{net} \propto 1/r$ . Therefore, the gravitational potential that causes this force must be logarithmic in r. The general form for a logarithmic dark matter halo potential is given in Equation 1.4.

$$\Phi_{halo} = v_{halo}^2 \ln \left( X_{GC}^2 + Y_{GC}^2 + \frac{Z_{GC}^2}{q^2} + d^2 \right)$$
(1.4)

The quantity q is a Z direction flattening parameter. Values of q < 1 represent oblate matter distributions while values of q > 1 represent prolate matter distributions. The quantity d is the halo length scale parameter. The total mass of the halo is contained within the quantity  $v_{halo}$ , the halo speed. Values of 75 km s<sup>-1</sup> <  $v_{halo} < 150$  km s<sup>-1</sup> are typical, and represent halos with masses between  $10^{10}$  and  $10^{12}$  M<sub>Sun</sub>. One additional immediate observation of the logarithmic halo is that, unlike most elementary potentials, it does not vanish at infinity. This implies that it is meaningless to discuss the "total mass" of the halo, we can only calculate the enclosed mass within a certain Galactocentric radius. Certainly the total mass of the Milky Way is not infinite, and this potential must break down at extremely large radii, but these cases will not be considered here.

### 1.3.7 Triaxial Halo

The logarithmic halo derived above, even with the Z-direction flattening q, is a special case of the restricted triaxial logarithmic halo. A restricted triaxial halo is one with flattenings in all three directions  $(q_1, q_2, \text{ and } q_z)$ , and a free angle  $\phi$  to rotate the halo axes away from the coordinate system axes. The functional form for this restricted triaxial potential, from Law and Majewski (2010), is given in Equation 1.5

$$\Phi_{halo} = v_{halo}^2 \ln \left( C_1 X_{GC}^2 + C_2 Y_{GC}^2 + C_3 X_{GC} Y_{GC} + (Z_{GC}/q_z)^2 + d^2 \right)$$
(1.5)

with

$$C_1 = \left(\frac{\cos^2 \phi}{q_1^2} + \frac{\sin^2 \phi}{q_2^2}\right)$$
(1.6)

$$C_{2} = \left(\frac{\cos^{2}\phi}{q_{2}^{2}} + \frac{\sin^{2}\phi}{q_{1}^{2}}\right)$$
(1.7)

$$C_3 = 2\sin\phi\,\cos\phi\left(\frac{1}{q_1^2} - \frac{1}{q_2^2}\right)$$
 (1.8)

It is sufficient to set one flattening value to unity (usually  $q_2$ ) because it is only the ratios of the flattenings that is physically significant. The angle  $\phi$  rotates the halo X axis counterclockwise away from the coordinate X' axis in the XY plane, and therefore there are combinations of flattenings and  $\phi$  that can lead to degenerate potentials.

In the absolute general case, all three halo axes can be rotated away from the coordinate axes. This is achieved by applying the Euler-angle rotation matrix

$$\begin{pmatrix} q_1 X_{GC} \\ q_2 Y_{GC} \\ q_z Z_{GC} \end{pmatrix} = \mathcal{M} \begin{pmatrix} X'_{GC} \\ Y'_{GC} \\ Z'_{GC} \end{pmatrix},$$

 $\mathcal{M} = \begin{pmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta\\ \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \cos\theta\cos\psi\\ \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \cos\theta\cos\psi \end{pmatrix},$ 

to the general logarithmic halo

$$\Phi_{halo} = v_{halo}^2 \ln \left( X_{GC}^{\prime 2} + Y_{GC}^{\prime 2} + Z_{GC}^{\prime 2} + d^2 \right).$$
(1.9)

The angles  $\theta$ ,  $\psi$ , and  $\phi$  are the xyz pitch-roll-yaw Euler rotation angles. The angle  $\theta$  rotates the halo Z axis away from the coordinate Z' axis. The angle  $\phi$  is the same as before, rotating the halo X axis away from the coordinate X' axis. The angle  $\psi$  is the roll angle of the halo along the new X axis. Note that this definition is not the same as that used to describe the specialized stream coordinate systems.

#### 1.3.8 NFW Halo

One additional dark matter halo model is that derived by Navarro, Frenk and White (1996). This halo was devised to model equilibrium galactic matter distributions consistent with  $\Lambda$ CDM cosmologies. The mass density as a function of Galactocentric radius  $\rho(r)$  is given by Equation 1.10

$$\rho_{halo} = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2} \tag{1.10}$$

where  $r_s$  is the halo scale length and  $\rho_s$  is the principle halo mass density. The potential that corresponds to this density is represented as

$$\Phi_{halo} = -\frac{r_s v_{c,max}^2}{0.216} \ln\left(\frac{1+\frac{\mathbf{r}}{\mathbf{r}_s}}{\mathbf{r}}\right),\tag{1.11}$$

where  $v_{c,max}$  is the maximum halo circular speed which, like the logarithmic halo, is representative of the halo mass, and the factor of  $\frac{1}{0.216}$  is used to put the potential in the system of units described earlier (Klypin et al. 1999).

This halo has the advantage of describing the mass density of a ACDM consistent mass distribution It has the disadvantage that it is inherently spherical. Flattenings along any of the coordinate axes must be introduced in mass density profiles, rather than analytically in a gravitational potential (Law, et al. 2005). It is for this reason that we will consider the NFW halo only in cases comparing results to spherical logarithmic halos.

## 1.4 Dwarf Models

With the background Galactic gravitational potentials established, we now shift our attention to the models that will be used to represent the globular clusters and dwarf galaxies that will be disrupted. While the Galaxy will be modeled by a static potential, dwarfs are modeled with individual particles that are populated within a density distribution. In this subsection, we will enumerate some of the more commonly used dwarf density distributions: the Plummer, King, Jaffe, Hernquist, and Dehnen models. These will be described for completeness. For the remainder of this Thesis, we will use the Plummer model for all dwarfs.

In all of the dwarf models desribed below, the quantity  $r_d$  denotes the distance from the center of the dwarf.

### 1.4.1 Plummer Model

The Plummer model (Plummer, 1911) is used to model the stellar distributions of globular clusters. It is defined by the following potential/density pair (Binney and Tremaine, 1987, p. 42):

$$\Phi_{\rm P}({\rm r_d}) = -\frac{{\rm M_{\rm P}}}{\sqrt{{\rm r_d^2 + {\rm a}^2}}} \tag{1.12}$$

$$\rho_{\rm P}(\mathbf{r}_{\rm d}) = \frac{3M_{\rm P}}{4\pi} \frac{\mathbf{a}^2}{(\mathbf{r}_{\rm d}^2 + \mathbf{a}^2)^{\frac{5}{2}}},\tag{1.13}$$

where  $M_P$  is the total mass and a is the Plummer scale radius (the approximate physical extent of the cluster).

#### 1.4.2 Isothermal Sphere

The isothermal sphere is a simple model where the thermal energy of the particles is in hydrostatic equilibrium with the self-gravity of the cluster. They are useful in modeling because they do not self-collapse in the way that a uniform density sphere would, but they are not preferred over Plummer models due to the mass distribution being unlike globular clusters and dwarf galaxies. Nonetheless, we will provide the potential/density pair for this model:

$$\Phi_{\rm IS}(r_{\rm d}) = 4\pi a^2 \rho_0 \ln\left(\frac{r}{a}\right) \tag{1.14}$$

$$\rho_{\rm IS}(\mathbf{r}_{\rm d}) = \rho_0 \left(\frac{\mathbf{r}}{\mathbf{a}}\right)^{-2}.$$
 (1.15)

### 1.4.3 King Models

The King models are a class of dwarf models that, in the words of Binney and Tremaine "resembles the isothermal sphere at small radii, ..., and is less dense than the isothermal sphere at large radii."

They do not, in general, have well-represented potential/density pairs, but instead are represented by an energy  $(\epsilon)$  distribution function

$$f_K(\epsilon) = \begin{cases} \rho_K(2\pi\sigma^2)^{\frac{-3}{2}}(e^{\epsilon/\sigma^2} - 1) & \epsilon > 0\\ 0 & \epsilon \le 0 \end{cases}$$

where  $\sigma$  is the energy dispersion (Binney and Tremaine, 1987, p. 232).

## 1.4.4 Jaffe Model

The Jaffe model (Jaffe, 1983) is a simple model intended for spherical globular clusters and dwarf galaxies. The potential/density pair for this model is

$$\Phi_{\rm J}(r_{\rm d}) = \frac{M_{\rm J}}{\rm a} \ln \left( \frac{r_{\rm d}}{r_{\rm d} + \rm a} \right) \tag{1.16}$$

$$\rho_{\rm J}(r_{\rm d}) = \frac{M_{\rm J}a}{4\pi r_{\rm d}^2 (r_{\rm d} + a)^2}.$$
(1.17)

## 1.4.5 Hernquist Model

The Hernquist model was introduced by Hernquist (1990) to address difficulties of the Jaffe model. According to Hernquist: "the distribution function derived from the Jaffe model deviates from that of the de Vaucouleurs  $R^{1/4}$  law at large negative energies."

The potential/density pair of this model is

$$\Phi_{\rm H}(r_{\rm d}) = -\frac{M_{\rm H}}{r_{\rm d} + a} \tag{1.18}$$

$$\rho_{\rm H}({\rm r_d}) = \frac{{\rm M_H}}{2\pi} \frac{{\rm a}}{{\rm r_d}} \frac{1}{({\rm r_d}+{\rm a})^3}. \tag{1.19}$$

#### 1.4.6 Dehnen Model

A generalization of the Jaffe and Hernquist models was formulated by Dehnen (1993). The potential/density pairs for this model are

$$\Phi_D(r_d) = \frac{M_D}{a} \times \begin{cases} -\frac{1}{2-\gamma} \left( 1 - \left(\frac{r_d}{r_d+a}\right)^{2-\gamma} \right) & \gamma \neq 2\\ \ln\left(\frac{r_d}{r_d+a}\right) & \gamma = 2 \end{cases}$$

$$\rho_D(r_d) = \frac{(3-\gamma)M_D}{4\pi} \frac{a}{r_d^{\gamma}(r_d+a)^{4-\gamma}}.$$
(1.20)

The models of Jaffe and Hernquist correspond to the  $\gamma = 2$  and  $\gamma = 1$  cases, respectively.

## 1.5 The Process of Tidal Disruption

The tidal interaction is a secondary effect of the force of gravity, illustrated by the following thought experiment (outlined by Ohanian, 1976, p. 26): consider a space ship orbiting the Earth, with a water droplet at its center. The gravitational force from the Earth on the bottom of the droplet is greater than the force on the top of the droplet. Due to the surface tension holding the droplet together, the droplet bulges in the direction connecting its center to the center of the earth. The deformation of the droplet is called tidal disruption.

Similarly, instead of a water droplet, consider a bound collection of self gravitating particles whose center is the origin of a coordinate system with the z-axis parallel to the radial line to the center of the Earth. The gravitational force from the Earth on a particle of mass m at  $\vec{r} = (0, 0, z)$  is

$$\vec{F} = -\frac{GM_{Earth}m}{\left(r_0 + z\right)^2}\hat{z} \tag{1.21}$$

where  $r_0$  is the distance from the center of the Earth to the origin of our coordinate system. For small values of z relative to the distance to the Earth, we can Taylor expand  $\vec{F}$  around z to obtain

$$\vec{F} \approx -\frac{GM_{Earth}m}{r_0^2}\hat{z} + \frac{2GM_{Earth}mz}{r_0^3}\hat{z} + O\left(z^2\right)\hat{z}$$
(1.22)

We see that the first term is simply the acceleration of the origin relative to the Earth. So, relative to the origin, the particle experiences a force

$$\vec{F}_{tidal} = \frac{2GM_{Earth}mz}{r_0^3}\hat{z}.$$
(1.23)

We notice that this force is repulsive from the origin, and depends on the displacement z. This confirms our earlier thought experiment that the water droplet (or group of particles) would bulge in the radial direction.

Another way to view the tidal interaction is with the tidal radius. The tidal radius is the distance that a particle needs to be from a spherically symmetric, bound collection of particles to become gravitationally unbound. The tidal radius depends on the total mass of the collection, as well as the distance from the collection to the gravitational source (the Earth, in the above example). As the collection orbits the source, the tidal radius would become larger at large distance from the source (few particles would lie outside it) while at small distance from the source it would become small, allowing more particles to leave the collection.

Once a particle has left the collection, it becomes gravitationally bound to the source and assumes an orbit that is similar to, but not the same as, the original collection. Specifically, the particle's total energy is smaller (larger) than the average for particles leading (trailing) the collection. The totality of particles leading and trailing the collection is the tidal stream. This leads to a small, yet important, theoretical difficulty: how can we model the orbit of the progenitor when all we have is the stream? The stream, by definition, its not composed of particles with constant energy, so it should not even be possible to fit an orbit to it.

Whether we are able to approximate the stream with an orbit depends on how well the kinematics of the stream agree with those of the orbit that generated it. To understand possible deviations, we will reconsider the collection of particles introduced above, at the moment of tidal disruption. As the collection passes through perigalacticon (at a distance R), the tidal radius shrinks to a minimum and particles become unbound. Consider a particle whose energy differs from the dwarf by an amount  $\delta E$ . For a given time, the particle's orbit can deviate from the parent orbit in two ways: distance from the Galactic center,  $\delta r$ , or total speed,  $\delta v$ . The relative scale of these deviations is given by Equation 1.24.

$$\delta E = \left(\frac{\partial E}{\partial v}\right) \delta v + \left(\frac{\partial E}{\partial r}\right) \delta r \tag{1.24}$$

If we assume a form for the energy of the particle  $E = \frac{1}{2}mv^2 + m\Phi$ , Equation 1.24 becomes

$$\delta E = mv\delta v + m\frac{\partial\Phi}{\partial r}\delta r \tag{1.25}$$

Defining a new quantity  $\delta \epsilon = \delta E/m$  we obtain

$$\delta\epsilon = v\delta v + \frac{\partial\Phi}{\partial r}\delta r \tag{1.26}$$

If we assume a simple form of the Galactic gravitational potential  $\Phi = GM_{gal}/r$ , perigalactic distances of 10 kpc, velocities  $v \approx 100$  km s<sup>-1</sup>, and a total Galactic mass of  $M_{gal} \approx 10^{12} M_{Sun}$ , we find that the  $\delta v$  prefactor is several orders of magnitude smaller than the  $\delta r$  prefactor. Therefore, we expect that a particle removed from the collection via tidal disruption will deviate from the orbit in distance more than in velocity.

To illustrate this effect, we will construct two models of tidal disruption: I) a globular cluster with mass  $M = 1 \times 10^5$  M<sub>Sun</sub> and radius a = 0.01 kpc and II) a large dwarf galaxy with mass  $M = 1 \times 10^8$  M<sub>Sun</sub> and radius a = 0.3 kpc. A test orbit is created with starting parameters  $(l, b, R, v_x, v_y, v_z) = (172^\circ, 54^\circ, 8.5 \text{ kpc}, -90 \text{ km s}^{-1}, -230 \text{ km s}^{-1}, -115 \text{ km s}^{-1})$ . We construct Plummer sphere models (Plummer, 1911) for each of the test cases and evolve them along the orbit using the *gyrfalcON* tool of the NEMO Stellar Dynamics Toolbox (Teuben, 1995). Figures 1.3 and 1.4 show the evolution of the models in Galactic latitude *b*, Galactic Standard of Rest radial velocity  $v_{qsr}$ , and Sun-centered distance  $d_{Sun}$ .

We see that an orbit is a good approximation for the stream of a cluster, while for the large dwarf galaxy, the orbit deviates in distance as was demonstrated



Figure 1.3 Model I: a globular cluster with mass  $M = 1 \times 10^5$  M<sub>Sun</sub> and radius a = 0.01 kpc. Top panel shows the test orbit and disruption in Galactic coordinates l and b, middle shows Galactic Standard of Rest radial velocity  $v_{gsr}$  vs. Galactic longitude l, and bottom shows Sun-centered distance  $d_{Sun}$  vs. Galactic longitude l. The model disruption agrees with the test orbit except near the stream ends, where torquing from the Galactic disk causes deviation from the orbit in sky coordinates. The stream can be well fit with an orbit for this case.


Figure 1.4 Model II: a large dwarf galaxy with mass  $M = 1 \times 10^8$  M<sub>Sun</sub> and radius a = 0.3 kpc. Top panel shows the test orbit and disruption in Galactic coordinates l and b, middle shows Galactic Standard of Rest radial velocity  $v_{gsr}$  vs. Galactic longitude l, and bottom shows Sun-centered distance  $d_{Sun}$  vs. Galactic longitude l. The model disruption does not agree with the test orbit in either velocity or distance. An orbit cannot be reliably fit to a stream of this type.

before. For a large dwarf galaxy, we also start to see deviation in the radial velocity. Therefore, modeling tidal streams with orbits is a good first-order approximation for globular cluster tidal streams. The negligible deviation in velocity allows us to use the velocities of the stream stars directly. The deviation in distance can potentially be dealt with in two ways. First, the distance estimates may be so poor that the orbit would be consistent with the stream. Secondly, the direction of the stream can be obtained with radial velocity measurements. One can then know whether the stream is a leading or trailing tail, and can adjust the measured distances to find the best fit orbit.

An additional effect that can be noticed from these models is the fact that the particles at the ends of the streams also deviate from the orbit in sky coordinates l and b. This is due to the non-spherical disk torquing the highest-energy particles at perigalacticon and causing displacement on the sky. We will see in Chapter 3 that mis-determination of stream quantities out near the ends of the stream can lead to significant modeling difficulties, and that errors in stream quantities need to be relaxed near stream ends.

While these models illustrate the difficulties with fitting orbits to tidal streams, they do not answer the underlying question: can we constrain the Galactic background by fitting an orbit to a stream? After a generalized fitting method is established in the next Chapter, this issue will be discussed in Chapter 3.

## CHAPTER 2 ORBIT FITTING OF TIDAL STREAMS

## 2.1 General Orbit Fitting Method

To develop an orbit fitting framework, it is necessary to understand that tidal streams are collections of stars that have been disrupted by Galactic tidal forces. It is therefore important to consider all of the stars within a stream. Suppose that we had the full 6-D phase space information for a single star within a stream. This complete description would be insufficient to establish an orbit for the stream, as it would instead establish the orbit of that particular star. While individual stream stars do not deviate significantly from the stream as a whole, we must endeavour to understand the statistical properties of the kinematics of all stream stars to construct a consistent orbit. Before proceeding, we must decide the ideal quantities to be used in fitting the stream.

#### 2.1.1 Stream Quantities

We seek to find an orbit that most closely corresponds to the collection of stream stars. But in what quantities should the correspondence be expected? As was demonstrated in the previous section, a stellar stream and its orbit closely correspond in sky position and radial velocity, but not necessarily in distance. This separation of coordinates lends itself to stream fitting. Suppose instead we chose to attempt to fit a stream in Cartesian coordinates. This would result in tremendous difficulty because a well known radial velocity gets transformed into three components, each with more uncertainty. Also, as will be addressed soon, distances to stars in the Galactic environment are less well known than other quantities, and these uncertainties would likewise become transformed into three components.

We can, in principle, be in possession of all phase space information of stream stars. This is accomplished by measuring the angular position on the sky (in any system, notated in general as two angles  $[\theta, \phi]$ ), the Sun-centered distance to the star (through various tracer populations to be described later), the Galactic Standard of Rest radial velocity  $(v_{gsr})$  of the star (derived from Doppler shift of stellar spectra), and the proper motion of the star on the sky.

The primary difficulty is that some of these quantities are more well known than others. Angular positions  $(\theta, \phi)$  can be measured with extreme accuracy. Radial velocities can be measured to levels of 5 km s<sup>-1</sup>. Distances and proper motions, however, are generally much less well known. Galactic distances typically have errors of one part in ten, and proper motions may have errors of near one hundred percent.

Therefore, even though orbits are computed and evolved in Galactocentric Cartesian coordinates, they are fit in the space of sky coordinates, radial velocities, and Sun-centered distances. We must disregard proper motions of stream stars, and attempt to find another way to determine the unknown velocity components.

#### 2.1.2 Distance Estimation

Distances to stream stars are amongst the least known quantities. Tidal streams are far enough away to escape distance estimation via parallax. In this subsection, we describe the general methods used to obtain stream distances. Special cases and considerations will be examined in discussions of real streams.

To estimate distances to tidal streams, we must rely on statistics to populate a color-magnitude diagram, and identify populations of stars with a known absolute magnitude. We make use of two particular stellar populations: F-turnoff stars, and Blue Horizontal Branch stars (BHBs). Shown in Figure 2.1 is a color-magnitude diagram (CMD) of the globular cluster Palomar 5 (Pal 5), showing the top of the main sequence, turnoff, giant branch, and horizontal branch in SDSS colors. The turnoff (located at  $20 < g^* < 21$  with colors  $0.1 < (g^* - r^*) < 0.3$ ) is a useful distance indicator because if a certain absolute magnitude is chosen, then the narrow range of apparent magnitudes yields an approximate distance. The starred magnitudes represent SDSS commissioning magitudes. More precise distance estimates can be obtained if an absolute magnitude distribution is assumed (e.g. Cole, 2008), and convolved with the stellar population. For our purposes, we will utilize a single SDSS absolute magnitude of  $M_{g_0} = 4.2$  and blue turnoff colors of  $0.0 < (g - r)_0 < 0.3$  and  $(u - g)_0 > 0.4$ . In Figure 2.1, we can also see the Horizontal Branch, located at  $-0.3 < (g^* - r^*) < 0.0$  and  $g^* = 17.5$ . These stars provide a good distance indicator because the Horizontal Branch is at an approximately constant apparent magnitude, bending toward higher magnitudes at the very blue end. The difficulty with this population is their relative sparseness compared to main sequence and turnoff stars. Additionally, it is necessary to separate the Horizontal Branch stars from Blue Stragglers (stars still on the main sequence but with colors of BHBs). Within the SDSS, BHBs are contained within the colors  $-0.3 < (g - r)_0 < 0.2$ , with an apparent magnitude approximately 3.5 mag brighter than the turnoff magnitude.

In either case, a color magnitude diagram is constructed for each stream location, and the apparent magnitude  $g_0$  is determined for the desired population (be it F-turnoff or Blue Horizontal Branch stars). The distance is determined by Equation 2.1,

$$g - M_g = 5 \log_{10} \left( \frac{\mathrm{d}}{10 \mathrm{\,pc}} \right) \tag{2.1}$$

where  $M_{g_0} = 4.2$  for F-turnoff (Cole, 2009) and  $M_g = 0.45$  for Blue Horizontal Branch stars (Newberg, et al., 2010).

#### 2.1.3 Determination of Initial Guess Velocity

To perform the gradient search method that will be outlined in the following sections, we require an initial guess of the stream velocity. Even without stellar proper motions, we can determine the direction of a stellar stream along the sky. Suppose we take two stream locations, that have  $(\theta, \phi, v_{gsr}, d_{sun}) = (\theta_1, \phi_1, v_1, d_1)$  and  $(\theta, \phi, v_{gsr}, d_{sun}) = (\theta_2, \phi_2, v_2, d_2)$ . Using these two stream locations, and a known value for the Sun-Galactic Center distance  $(R_{sun} = 8.0 \text{ kpc})$ , we convert these locations into right-handed Galactic XYZ coordinates  $\vec{r_1} = (X_1, Y_1, Z_1)$  and  $\vec{r_2} = (X_2, Y_2, Z_2)$ . Subtracting these gives the direction vector of the stream  $\vec{\Delta} = (X_1 - X_2, Y_1 - Y_2, Z_1 - Z_2)$ . Since the stream goes from Point 1 to Point 2, the velocity of the stream points in the direction of  $\vec{\Delta}$ , namely  $\vec{v} = v_{mag}\hat{\Delta}$ . It is also constrained by the Galactic Standard of Rest radial velocity at Point 1 ( $v_{gsr,1}$ ). Therefore we can solve for the velocity magnitude  $|\vec{v}_{mag}| = \frac{v_{gsr,1}}{\vec{r_1}\cdot\vec{\Delta}}$ . We will therefore fit streams in



Figure 2.1 Color Magnitude Diagram (CMD) of globular cluster Pal 5 in SDSS colors from Yanny & Newberg et al. (2000). The turnoff is shown between  $20 < g^* < 21$  with colors  $0.1 < (g^* - r^*) < 0.3$ . The stars indicate SDSS commissioning magnitudes. Additionally, a Horizontal branch is shown as circled points at  $g^* = 17.5$  with colors  $-0.3 < (g^* - r^*) < 0$ . Care must be taken to avoid Blue Stragglers, which are shown as circled points at  $19 < g^* < 20$  and  $-0.2 < (g^* - r^*) < 0$ . These two populations are useful distance indicators, despite their intrinsic variability in brightness. They provide sufficient distance constraints to be used in model orbit fitting. Figure reproduced with permission.

the following coordinates: the sky coordinates of stream location  $(\theta, \phi)$ , the velocity  $\vec{v} = (v_x, v_y, v_z)$ , and the Sun-centered distance R.

#### 2.1.4 Fit Parameters

With the fit quantities established, we must now consider the parameters that influence those quantities. We pick a stream location, whose kinematics are determined by six quantities:  $(\theta, \phi, R, v_x, v_y, v_z)$ . The coordinate system of the velocity components is arbitrary. We could very well choose a system that includes the radial velocity  $v_{gsr}$ , and two tangential velocities  $v_t$  and  $v_u$ . We choose the Galactic velocity components for simplicity. Since the sky positions  $(\theta, \phi)$  of a stream location are well known, these are not parameters to be fit. The other quantities:  $(R, v_x, v_y, v_z)$  are to be fit. Additionally, any parameters contained within the previously defined Galactic potentials may be fit.

#### 2.1.5 Construction of a Fitness Function

With the relevant stream quantities and parameters determined, we now define a fitness metric. Let the vector  $\vec{Q} = (R, v_x, v_y, v_z, ...)$  describe the parameters to be fit. For each  $\vec{Q}$  we seek to create a fitness metric  $\chi^2(\vec{Q})$ . As mentioned previously, the stream quantities to be fit are the sky locations  $(\theta, \phi)$ , radial velocities (rv), and Sun-centered distances (R). We take the sky coordinate  $\theta$  as an independent variable, and construct three goodness of fit metrics for the remaining quantities.

$$\chi_{\phi}^{2} = \sum_{i} \left( \frac{\phi_{model,i} - \phi_{data,i}}{\sigma_{\phi}} \right)^{2}$$
(2.2)

$$\chi_{rv}^2 = \sum_{i} \left( \frac{rv_{model,i} - rv_{data,i}}{\sigma_{rv,i}} \right)^2 \tag{2.3}$$

$$\chi_R^2 = \sum_i \left(\frac{R_{model,i} - R_{data,i}}{\sigma_R}\right)^2 \tag{2.4}$$

These three metrics are minimized when the orbit and stream are coincident in all three quantities. To combine the metrics into a single fitness, we simply add them and normalize:

$$\chi^{2}_{stream} = \frac{1}{\eta} \left( \chi^{2}_{\phi} + \chi^{2}_{rv} + \chi^{2}_{R} \right), \qquad (2.5)$$

where  $\eta = N - n - 1$ , N is the number of data points, and n is the number of parameters.

To calculate these  $\chi^2$  values, we calculate a model orbit using the selected parameters. We search the orbit for two  $\theta_{model}$  values, one on each side of the data point  $\theta_{data}$ . We linearly interpolate the orbit via Equations 2.6 through 2.8, and use the associated  $\phi_{model}$ ,  $rv_{model}$  and  $R_{model}$  to compute  $\chi^2$ .

$$\phi_{model,i} = \frac{\phi_{model,k+1} - \phi_{model,k}}{\theta_{model,k+1} - \theta_{model,k}} \left(\theta_{data,i} - \theta_{model,k}\right) + \phi_{model,k} \tag{2.6}$$

$$rv_{model} = \frac{rv_{model,k+1} - rv_{model,k}}{\theta_{model,k+1} - \theta_{model,k}} \left(\theta_{data,i} - \theta_{model,k}\right) + rv_{model,k}$$
(2.7)

$$R_{model,i} = \frac{R_{model,k+1} - R_{model,i}}{\theta_{model,k+1} - \theta_{model,k}} \left(\theta_{data,i} - \theta_{model,k}\right) + R_{model,k}$$
(2.8)

The total fitness of the model depends on more than just the stream. If we are fitting parameters of the background gravitational potential, we must also ensure that the simulated Galactic rotation curve is consistent with observations. For this work, we adopt the Simulation 1 measurements obtained by Xue, et al. (2008), which are shown in Figure 1.2. To construct a fitness for the rotation curve, we simply calculate the rotation speed due to the Galactic potential at the radii measured by Xue, and sum the squared differences between the model rotation curve and the data,

$$\chi^2_{rotcurve} = \frac{1}{\eta_{rotcurve}} \sum_i \left(\frac{v_{model,i} - v_{data,i}}{\sigma_v}\right)^2 \tag{2.9}$$

where  $\eta_{rotcurve} = N_{rotcurve} - n_{rotcurve} - 1$ , where  $N_{rotcurve}$  is the number of rotation curve data points and  $n_{rotcurve}$  is the number of Galactic potential parameters being fit.

The combined fitness value we then take as the average of the stream fitness and the rotation curve fitness,

$$\chi^2 = \frac{\chi^2_{stream} + \chi^2_{rotcurve}}{2} \tag{2.10}$$

For the case of multiple streams, the combined fitness extends trivially. If M is the number of streams, the fitness becomes

$$\chi^2 = \frac{\left(\sum_i^M \chi^2_{stream,i}\right) + \chi^2_{rotcurve}}{M+1}$$
(2.11)

#### 2.1.5.1 Distance Scale Factors

The stream fitness metric defined above is minimized when the orbit distance precisely matches the stream distance for a particular value of  $\theta$ . As was shown

previously, the tidal stream generated by evolution on a specific orbit need not match that orbit in distance due to energy segregation. There are several ways to remedy this difficulty. The first is to not fit the stream distances at all (Law and Majewski, 2010). This method can be effective if the progenitor of a tidal stream is well known, as the orbit must pass through said progenitor and it can constrain the distance scale. However, in the case of no known progenitor, we can still fit the distances to the tidal stream by introducing distance scale factors  $S_{leading}$  and  $S_{trailing}$ . These are multiplicative factors are applied to the stream distance data to scale it closer or farther from the Sun in an attempt to have the orbit pass through the stream in distance. We expect the distance scale factors to have values  $S_{leading} > 1$  and  $0 < S_{trailing} < 1$ . To ensure these constraints are met, we introduce two additional positive-definite parameters  $\sigma_{leading}$  and  $\sigma_{trailing}$  such that if  $\sigma_{leading} > 1$  then  $S_{leading} = \sigma_{leading}$  and if  $0 < \sigma_{leading} < 1$  then  $S_{leading} =$  $1 + \sigma_{leading}$ . Similarly, if  $\sigma_{trailing} < 1$  then  $S_{trailing} = \sigma_{trailing}$  and if  $\sigma_{trailing} > 1$  then  $S_{trailing} = \sigma_{trailing} - int(\sigma_{trailing})$ . The parameters  $\sigma_{leading}$  and  $\sigma_{trailing}$  can thus be fit, and appropriate values of  $S_{leading}$  and  $S_{trailing}$  can be guaranteed. These scale factors are only used in cases where we fit debris from large dwarf galaxies such as the Sagittarius Dwarf. Hereforth, where they are used, they will be mentioned.

#### 2.1.6 Finding the Best Fit Solution

With a fitness metric defined, we now optimize the orbital parameters such that  $\chi^2$  is minimized. To do this, we choose an initial set of parameters, calculate an orbit within a Galactic potential of our choosing, and calculate  $\chi^2$ . We can minimize  $\chi^2$  using a variety of search methods. The two used primarily in this Thesis are the gradient search method, and the particle swarm optimization method.

#### 2.1.6.1 Gradient Search

The gradient search method is one which analyzes a parameter space by moving along the direction of steepest descent towards a minimum value. As introduced before, let the vector  $\vec{Q} = (R, v_x, v_y, v_z, ...)$  describe the orbital parameters at the desired starting location, as well as the halo parameters to be fit. For each  $\vec{Q}$ , there is an associated  $\chi^2(\vec{Q})$ . We choose an initial set of parameters  $\vec{Q_0}$  and find  $\chi^2(\vec{Q_0})$ . We then iterate the parameters using

$$Q_{i,new} = Q_{i,old} - h_i \Lambda \vec{\nabla}_i \chi^2 \left( \vec{Q_{old}} \right).$$
(2.12)

We calculate the gradient using a finite-difference method,

$$\nabla_i \chi^2(\vec{Q}) \approx \frac{\chi^2(Q_i + h_i) - \chi^2(Q_i - h_i)}{2h_i} \bigg|_{\text{all other } Q_k \text{ fixed.}}$$
(2.13)

Different values of  $h_i$  are used because the parameters are on different scales, it would not be appropriate to use the same step size for them all. Table 2.1 shows the step sizes used for specific quantities.

Quantity	Step Size, h
Distances	$0.1 \; \mathrm{kpc}$
Velocities	$1 \ {\rm km} \ {\rm s}^{-1}$
Flattenings	0.01
Angles	$1^{\circ}$

Table 2.1 Step sizes of quantities used in gradient search and Hessian error estimation

 $\Lambda$  is a variable-learning parameter. It initially begins at  $\Lambda = 1$ , and if the new value of  $\chi^2(\vec{Q})$  is smaller than the old, then  $\Lambda$  is multiplied by 1.03, if not, it is multiplied by 0.80. The purpose of this is to ensure if a minimum is being found, then it is found faster than with a constant-learning parameter. We also multiply it by the associated  $h_i$  value to make the step size appropriate for the parameter being considered.

#### 2.1.6.2 Particle Swarm Optimization

Particle swarm optimization, first devised by Kennedy and Eberhart (1995), is an optimization method inspired by biological systems such as flocking birds and schools of fish. As described by Desell (2009): "This approach consists of a population of particles, which fly through the search space based on their previous velocity, their individual best found position (cognitive intelligence) and the global best found position (social intelligence)." A collection of "particles" are created by generating parameters within userdefined ranges and evaluating a  $\chi^2$  value for each of these parameter sets. The particles are assigned "velocities" through the parameter space, and the particles evolve according to

$$v_{\vec{Q}_{i}}(t+1) = w_{i}v_{\vec{Q}_{i}}(t) + c_{1}rand()\left(\vec{Q}_{p,i} - \vec{Q}_{i}\right) + c_{2}rand()\left(\vec{Q}_{g} - \vec{Q}_{i}\right)$$
(2.14)

and

$$\vec{Q}_i(t+1) = \vec{Q}_i + v_{\vec{Q}_i}(t+1).$$
(2.15)

In this formulation,  $\vec{Q}_{p,i}$  denotes the parameters corresponding to a particle's personal best  $\chi^2$  value, while  $\vec{Q}_g$  denotes the parameters corresponding to the global best  $\chi^2$  value. The parameters  $w_i$ ,  $c_1$  and  $c_2$  are user defined constants. The values of these constants determine the behavior of the search algorithm (Montes de Oca, 2007). The default values for these parameters in this Thesis are  $w_i = 0.5$ ,  $c_1 = 1.0$ and  $c_2 = 1.0$ . Any deviation from these values will be noted. This search method will become useful when optimizing over a large number of tidal streams and Galactic potential parameters.

#### 2.1.6.3 Parameter Error Estimation

As described by Cole (2008): "[parameter] accuracy depends upon the shape of the likelihood surface at its [minimum]." In a manner similar to Cole, and described by Willett, et al. (2009), we utilize a Hessian method to estimate parameter errors for our searches. We construct a matrix  $\mathbf{V}$  of second partial derivatives of the  $\chi^2$ surface, evaluated at the minimum found by the search method. The step sizes are the same used in the gradient search, given in Table 2.1. The error estimate for the  $i^{th}$  parameter is  $\sigma_i = \sqrt{2V_{ii}}$ . The  $\mathbf{V}$  matrix is defined as

$$\mathbf{V} \equiv \mathbf{H}^{-1}.\tag{2.16}$$

The matrix  $\mathbf{H}$  is the Hessian Matrix, whose elements are given by

$$H_{ij} = \frac{H_{ij}^{1} - H_{ij}^{2} - H_{ij}^{3} + H_{ij}^{4}}{4h_{i}h_{j}}, \text{ where,}$$
(2.17)  

$$H_{ij}^{1} = \chi^{2}(Q_{j} + h_{j}, Q_{i} + h_{i})|_{\text{all other } Q_{k} \text{ fixed}}$$
  

$$H_{ij}^{2} = \chi^{2}(Q_{j} - h_{j}, Q_{i} + h_{i})|_{\text{all other } Q_{k} \text{ fixed}}$$
  

$$H_{ij}^{3} = \chi^{2}(Q_{j} + h_{j}, Q_{i} - h_{i})|_{\text{all other } Q_{k} \text{ fixed}}$$
  

$$H_{ij}^{4} = \chi^{2}(Q_{j} - h_{j}, Q_{i} - h_{i})|_{\text{all other } Q_{k} \text{ fixed}}$$
  
(2.17)

# CHAPTER 3 VERIFYING ORBIT FITS WITH SIMULATED TIDAL STREAMS

With the orbit fitting method of the previous chapter established, we now move to verify its effectiveness by fitting simulated tidal streams. If we were to immediately apply this method to real streams, but were unable to fit them, there would be an ambiguity: does our fitting method not work, or are the underlying Galactic potentials unsuitable for modeling real streams? We can alleviate the first concern by creating simulated tidal streams within known Galactic potentials, and attempting to fit them to recover the parameters.

In this chapter, we will consider a variety of simulated situations. The first will include a single stream with unknown kinematics, followed by consideration of a single stream with unknown kinematics and potential parameters. We will comment on the constraints placed on the total halo mass via fitting in an incorrect halo model. Finally, we will address fitting multiple streams in axisymmetric and triaxial halo models.

The simulated tidal streams in this chapter are specifically chosen to mimic the real tidal streams that will be discussed in the next chapter. We will therefore only draw conclusions that are based upon these particular examples. The ultimate purpose of this chapter is to establish whether, even in principle, simulated tidal streams similar to those to be discussed can place constraints on the Galactic gravitational potential. The first simulated stream is chosen to mimic the Stream of Grillmair & Dionatos (GD-1), the second the Orphan Stream, and the third the Sagittarius Dwarf Tidal Stream.

## 3.1 Stream Generation

A simulated stream is generated by choosing a set of kinematic parameters  $\vec{Q} = (\theta, \phi, R, v_x, v_y, v_z)$  and an underlying Galactic potential, and evaluating an orbit. The orbit time must be sufficiently long to ensure significant disruption,



Figure 3.1 Illustration of stream generation process in Galactic Cartesian coordinates. A set of kinematic parameters is chosen and an orbit evolved back in time. A Plummer sphere, shown in blue, is placed at the location and velocity predicted by the orbit, and evolved forward in time to create a disrupted stream, shown in red. The forward direction of the orbit is indicated by the arrow.

but not so long as to violate the assumption of a static Galactic potential. These simulations adopt an orbit time of  $t_{back} = 4$  Gyr. All of our orbits are created and evaluated using the *mkorbit* and *orbint* tools of the NEMO Stellar Dynamics Toolbox (Teuben, 1995).

With an orbit that predicts the kinematics for our model at a time 4 Gyr in the past, we place a Plummer sphere with N = 10000 particles at this location and velocity, and evolve it forward in the same Galactic potential used to generate the orbit. The Plummer sphere parameters a and  $M_P$  are dependent on the type of stream we wish to generate. The N-body evolution code used for this chapter is the gyrfalcON tool (Dehnen, 2002) of the NEMO Stellar Dynamics Toolbox (Teuben, 1995). This sequence of events is depicted in Figure 3.1.

With the stream realized, we now disregard the real orbit used to create it,

and analyze the simulated stream itself. We choose to represent the stream in the  $(l, b, v_{gsr}, d_{Sun})$  system for reasons described in the previous chapter. This choice is arbitrary. We could very well choose to represent the stream in any angle and velocity space, so long as proper conversions are done. We select a set of l windows  $2^{\circ}$  wide, and find the mean  $b, v_{gsr}$ , and  $d_{Sun}$  values predicted by the stream, as well as the errors of these quantities. To ensure that we are only analyzing real stream particles, as opposed to outliers, we use a  $2\sigma$  clipped mean.

#### 3.1.1 Distance Estimation

The errors in distance require special consideration. As described previously, distances are determined by examining the magnitude of F-turnoff and Blue Horizontal Branch stars. Distances are thus limited by the uncertainty in the absolute magnitudes of these stellar populations. In a simulated stream, we know the distances to the particles exactly. We must therefore perturb the simulated stream distances to make them representative of a true stream.

Cole (2008) utilized an F-turnoff magnitude of  $4.2 \pm 0.6$  mag. We convert our stream distances to magnitudes using Equation 2.1. The magnitudes are then perturbed by a Gaussian random number with mean zero and sigma of 0.6 mag, and converted back to distances. To illustrate this effect, Figure 3.2 shows a simulated stream in  $(l, d_{Sun})$  space. The left panel shows the distances to the simulated stream, while the right panel shows the same stream with perturbed distances. We then analyze the perturbed distances with a 2  $\sigma$  clipped mean to obtain distance means and errors.

#### 3.2 Fitting a Single Simulated Stream

With a stream generation method established, we now consider fitting a single simulated stream under under a variety of conditions. These conditions include: known and unknown halo potential parameters, with and without supplementary rotation curve data, and in a similar yet incorrect halo model.



Figure 3.2 Illustration of simulated stream distance perturbation. Left panel shows the Sun-centered distance of a simulated globular cluster tidal stream. The F-turnoff magnitudes of the stream particles have been perturbed by a Gaussian random number with mean zero and sigma of 0.6 mag in the right panel.

#### 3.2.1 Single Simulated Stream: Known Background Parameters

The first situation we will consider is a single simulated stream evolved within a known Galactic potential. The stream's simulated orbit has initial kinematics of  $(l, b, R, v_x, v_y, v_z) = (170^\circ, 50^\circ, 10 \text{ kpc}, -100 \text{ km s}^{-1}, -300 \text{ km s}^{-1}, -100 \text{ km s}^{-1})$ . It is evolved for 4 Gyr within a Bulge + M-N Disk + Logarithmic halo with parameters given in Table 3.1. The disk and bulge models and parameters will remain constant throughout this entire analysis. The cluster is represented by a Plummer model with radius a = 0.2 kpc and mass  $M_P = 10 M_u \approx 2 \times 10^7 M_{Sun}$ . The clipped mean detections are given in Table 3.2 and the stream depicted in Figure 3.3.

We use the  $l = 175^{\circ}$  and  $l = 225^{\circ}$  points and the method described in Chapter 2 to establish a velocity guess of  $v = (-50 \text{ km s}^{-1}, -180 \text{ km s}^{-1}, -70 \text{ km s}^{-1})$ . Gradient searches were performed on random parameter sets by taking the principle values,  $(R, v_x, v_y, v_z) = (9.8 \text{ kpc}, -50 \text{ km s}^{-1}, -180 \text{ km s}^{-1}, -70 \text{ km s}^{-1})$ , and perturbing them by random numbers between  $\pm 75$  percent of their values. Seven

Parameter	Value
$M_{\rm disk}$	$1.0 \times 10^{11} \mathrm{M}_{\mathrm{Sun}}$
$M_{\mathrm{bulge}}$	$3.4 \times 10^{10} \mathrm{M}_{\mathrm{Sun}}$
$a_{\rm disk}$	$6.5 \ \mathrm{kpc}$
$b_{\rm disk}$	$0.26 \ \mathrm{kpc}$
$c_{\mathrm{bulge}}$	$0.7 \ \mathrm{kpc}$
$v_{ m halo}$	$114 {\rm ~km~s^{-1}}$
q	1.0
d	$12.0 \ \mathrm{kpc}$

Table 3.1 Fixed Galactic potential parameter values

l (°)	b(°)	$\delta b~(^\circ)$	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} (\mathrm{km} \mathrm{s}^{-1})$	$d_{Sun} (kpc)$	$\delta d_{Sun} (kpc)$
100	42.59	0.12	-207.9	0.5	15.5	0.7
125	50.73	0.14	-157.8	0.6	11.6	0.8
150	52.05	0.23	-102.2	0.9	10.5	0.5
175	49.09	0.22	-32.3	0.6	10.9	0.6
200	39.43	0.22	57.5	0.9	9.8	0.4
225	16.71	0.21	172.5	0.9	11.5	0.3

Table 3.2 Single stream dataset for spherical logarithmic halo with  $v_{halo} = 115 \text{ km s}^{-1}$  and d = 12 kpc.

gradient searches were performed, two reached local minima, while five reached a global minimum. The average of the five good searches, including parameter errors, is shown in Table 3.3 . The fitness value of this solution is  $\chi^2 = 5.52$ . The high value of this fitness is not cause for alarm. The small velocity errors determined by the clipped mean give rise to large  $\chi^2$  values due to the fact that orbit does not precisely match the stream. If the velocity errors are increased to 5 km s<sup>-1</sup>, the fitness value drops to  $\chi^2 = 1.60$ . This issue will be further addressed in the next section. The best fit and real orbits, as well as the stream detections are shown in Figure 3.4. We can see that the fit orbit is consistent with the true orbit. The only deviation occurs at large distances, but as was shown in the previous chapter, this is not an unexpected effect.

While the gradient search is able to find a solution to this stream that is consistent with the real orbit, it made apparent that this search method is prone to finding local minima. We attempt to alleviate this difficulty by conducting random parameter starts, however, as the number of parameters increases, gradient search



Figure 3.3 Simulated tidal stream in logarithmic halo with  $v_{halo} = 115$  km s<sup>-1</sup>, q = 1.0 and d = 12 kpc. Top panel shows stream and true orbit in (l, b) sky coordinates. Middle shows stream radial velocity  $v_{gsr}$  as a function of l, while bottom shows perturbed stream distance  $d_{Sun}$  as a function of l.

Parameter	Value
R	$9.8\pm0.6~{ m kpc}$
$V_x$	$-97\pm2~{\rm km~s^{-1}}$
$V_y$	$-300 \pm 2 \text{ km s}^{-1}$
$V_Z$	$-103 \pm 2 \text{ km s}^{-1}$
$\chi^2$	5.52

Table 3.3 Best gradient search fit of kinematic parameters parameters of a single simulated stream.

will soon become an ineffective search method.

For contrast, we will now analyze the same single simulated stream using particle swarm optimization. The aim of this is to see if, given the same data and starting information, particle swarm is a more effective minimization method. Utilizing particle swarm optimization with 5 particles, we find the best fit solution given in Table 3.4. The fitness of this solution is  $\chi^2 = 5.52$ . We can see that particle swarm was able to find the same solution, without converging to any local minima. The disadvantage is particle swarm required 2546 function evaluations, as opposed



Figure 3.4 Top panel shows true and fit orbits in (l, b) coordinates, middle panel shows  $(l, v_{gsr})$ , and bottom panel  $(l, d_{Sun})$ . It can be seen that the fit orbit is a very good approximation of the true orbit. The only deviation can be seen at low and high l values in radial velocity and distance. The previous chapter showed that a stream can deviate from its orbit in distance near apogalacticon. This difference between true and fit is therefore not unexpected.

to the 1380 required by gradient search. The number of function evaluations and particles required will increase dramatically as the number of parameters increases but, as will be demonstrated later, the ability to find global minima will far outweigh the increase in function evaluations.

Parameter	Value
R	$9.8\pm0.6~{ m kpc}$
$V_{x}$	$-97\pm2~{\rm km~s^{-1}}$
$V_y$	$-300 \pm 2 \text{ km s}^{-1}$
$V_{Z}$	$-103\pm2~\mathrm{km~s^{-1}}$
$\chi^2$	5.52

Table 3.4 Best particle swarm fit of kinematic parameters parameters of a single simulated stream.

#### 3.2.2 Single Simulated Stream: Unknown Background Parameters

The next situation we will consider is fitting the kinematic and potential parameters of a single simulated stream. The purpose of this test is to determine whether a single stream can recover the parameters of the potential in which it is produced. We utilize the same simulated stream from the previous section. In addition to fitting the stream kinematics, we also fit the halo parameters  $v_{\text{halo}}$ , q, and d.

#### 3.2.2.1 Fitting Without Rotation Curve

Fitting the potential introduces two situations: one where we fit only the stream, and the other where we fit both the stream and the rotation curve of the potential. We consider the first case here, and the second in the next section.

Fitting the potential without a rotation curve simply requires introducing more parameters into the fit. We will utilize  $v_{\text{halo}}$ , q, and d starting values of 100 km s<sup>-1</sup>, 1.0 and 12 kpc, respectively. Results from five random gradient search starts are given in Table 3.5. The Hessian calculation for this fit did not converge, therefore no parameter errors are reported.

R (kpc)	$v_x (km \ s^{-1})$	$v_y \ (km \ s^{-1})$	$v_z \ (km \ s^{-1})$	$v_{halo} \; (km \; s^{-1})$	q	d (kpc)	$\chi^2$
9.5	-86	-274	-98	11	14.9	20.0	6.38
9.5	-86	-274	-98	14	3.09	6.1	6.11
9.5	-86	-274	-98	13	2.38	9.0	6.23
9.5	-87	-275	-98	54	9.13	27.7	5.65
9.5	-87	-274	-98	32	19.5	26.1	5.97

Table 3.5 Results of five random gradient searches of a single simulated stream with potential parameters, without rotation curve.

The results of Table 3.5 demonstrate that, in general, we are unable to constrain the potential with this single stream without a rotation curve.

#### 3.2.2.2 Fitting With Rotation Curve

We now consider the case where we attempt to fit the kinematic and halo parameters of a single stream with a rotation curve. Before we can proceed, we must comment on the generation of the simulated rotation curve. For a given Galactic potential, the rotation curve is well defined for a specific set of potential parameters. Therefore, if we generate a rotation curve, and fit the parameters associated with it, there will be a well-defined, perfect fit. In a manner similar to the stream distances, we must perturb the rotation curve velocities to account for experimental errors. We generate a rotation curve for the perfect halo, and perturb each velocity by a Gaussian random number with mean zero and sigma 20 km s<sup>-1</sup>. This is an error representative of the Xue, et al. (2008) rotation curve data.

In the manner described in Chapter 2, we fit a single stream within a logarithmic halo with a sample rotation curve. We fit via gradient search with five random starts. The kinematic results of these starts, with Hessian errors, are shown in Table 3.6, while the potential parameter results are given in Table 3.7.

N	R (kpc)	$v_x (km \ s^{-1})$	$v_y \ (km \ s^{-1})$	$v_z \; (km \; s^{-1})$
1	$9.7\pm0.7$	$-96 \pm 5$	$-297 \pm 12$	$-104 \pm 4$
2	$9.5\pm0.6$	$-89 \pm 2$	$-279 \pm 3$	$-98 \pm 2$
3	$9.5\pm0.7$	$-89 \pm 2$	$-280 \pm 4$	$-98 \pm 2$
4	$9.6\pm0.6$	$-94 \pm 4$	$-292 \pm 10$	$-102 \pm 3$
5	$9.6\pm0.6$	$-92 \pm 4$	$-289 \pm 9$	$-101 \pm 3$

Table 3.6 Kinematic results of five random gradient searches of a single simulated stream with potential parameters and rotation curve.

N	$v_{halo} \ (km \ s^{-1})$	q	d (kpc)	$\chi^2$
1	$96 \pm 23$	$0.82 \pm 0.12$	$8.1\pm7.8$	5.4
2	$96 \pm 23$	$3.89\pm0.83$	$8.2\pm4.7$	8.2
3	$84 \pm 30$	$2.60 \pm 1.71$	$23.5\pm8.4$	8.5
4	$103 \pm 27$	$0.85\pm0.15$	$15.7\pm6.8$	5.3
5	$105 \pm 26$	$0.86 \pm 0.18$	$18.9\pm6.5$	5.5

Table 3.7 Halo parameter results of five random gradient searches of a single simulated stream with potential parameters and rotation curve.

Two particularly interesting observations can be made from these results. The first is the halo parameters are not at all recovered by these fits. The second is that there are two distinct minima recovered by the gradient search. This is problematic because if we can recover two distinct minima with five gradient searches, we have no way of knowing if the better of the two is the global minimum. To remedy this difficulty, we seek a global minimum via particle swarm optimization. Using N = 10 particles, we find best fit parameters shown in Table 3.8. The  $\chi^2$  for this fit is 3.98. We see that the lesser of the two fitness values found by gradient search is not a global minimum. We also see more reasonable halo parameters with this fitness. This conclusively demonstrates that particle swarm is a more effective and reliable search method for this problem.

Parameter	Value
R	$9.7\pm0.9~{ m kpc}$
$V_x$	$-100 \pm 6 \text{ km s}^{-1}$
$v_y$	$-308 \pm 14 \; \rm km \; \rm s^{-1}$
$V_{Z}$	$-107 \pm 5 {\rm ~km~s^{-1}}$
V <sub>halo</sub>	$123 \pm 22 \text{ km s}^{-1}$
q	$0.80\pm0.11$
d	$9.7\pm7.2~{ m kpc}$
$\chi^2$	3.98

Table 3.8 Best particle swarm fit of kinematic and potential parameters of a single simulated stream.

We now wish to see whether the particular perturbation of our rotation curve has an effect on the fit halo parameters. Since gradient search has proven to be unreliable, we perform particle swarm optimizations using four different rotation curve perturbations. The results, shown in Table 3.9 shows the halo parameters between different rotation curve perturbations are consistent with each other to within errors.

Rotation Curve	$v_{halo}$	q	d
1	$108 \pm 26$	$0.87\pm0.19$	$12.8 \pm 7.1$
2	$102 \pm 26$	$0.84\pm0.17$	$11.2 \pm 10.1$
3	$121 \pm 23$	$0.81\pm0.12$	$12.3\pm6.4$
4	$117\pm26$	$0.82\pm0.14$	$12.8\pm9.3$

Table 3.9 Best fit particle swarm halo parameters of a single simulated stream with potential parameters and different rotation curve perturbations.

We observe a slight discrepancy between the fit halo flattening q and the true value. While the fit and true values are consistent to within  $2\sigma$  confidence, the nature of a global minimum at  $q \approx 0.85$  deserves investigation. There are two

distinct possibilities: either the orbit of this stream is fundamentally unable to provide the correct value of q, or the errors of the stream quantities preclude a good fit.

To examine the first case, we imagine that the stream's orbit is all that is seen on the sky. We process the orbit like we would the stream, by selecting 2° wide slices in l and obtaining orbit values of  $b, v_{gsr}$ , and  $d_{Sun}$  using a clipped mean. These values can then be fit in the same manner as the simulated stream above. The errors on the orbit quantities are thus very small, and the orbit dataset is given in Table 3.10.

l (°)	$b(^{\circ})$	$\delta b$ (°)	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} (\mathrm{km \ s^{-1}})$	$d_{Sun} \; (kpc)$	$\delta d_{Sun} (kpc)$
100	42.659	0.006	-208.42	0.02	15.15	0.09
125	50.479	0.003	-158.78	0.04	11.81	0.10
150	52.289	0.001	-100.66	0.05	10.38	0.09
175	48.845	0.005	-31.60	0.07	9.94	0.09
200	38.429	0.011	59.58	0.07	10.31	0.08
225	17.545	0.013	170.16	0.05	12.62	0.06

Table 3.10 Orbit dataset for spherical logarithmic halo with  $v_{halo} = 115 \text{ km s}^{-1}$  and d = 12 kpc.

Fitting the orbit data using particle swarm optimization gives the best fit parameters shown in Table 3.11. These results are consistent with the true values at the  $2\sigma$  level. Fitting without the distance perturbation method gives the best fit parameters of Table 3.12. Finally, fitting the orbit without distance perturbations and with a perfect rotation curve gives the best fit values of Table 3.13.

Parameter	Value
R	$10.06\pm0.38\;{\rm kpc}$
$V_{x}$	$-101.5\pm0.8~{\rm km~s^{-1}}$
$V_y$	$-303.5\pm1.6~{\rm km~s^{-1}}$
Vz	$-100.8\pm0.5\;{\rm km\;s^{-1}}$
$v_{halo}$	$121.5 \pm 4.8 \text{ km s}^{-1}$
q	$0.97\pm0.03$
d	$11.8\pm0.5~{ m kpc}$

Table 3.11 Best particle swarm fit of kinematic and potential parameters of a single simulated stream orbit data with distance perturbation.

Parameter	Value
R	$10.000 \pm 0.001 \; \rm kpc$
$V_{\rm X}$	$-99.9 \pm 0.2 \text{ km s}^{-1}$
$v_y$	$-299.9 \pm 0.4 \text{ km s}^{-1}$
$V_{z}$	$-100.0\pm0.2~{\rm km~s^{-1}}$
$v_{halo}$	$111.9 \pm 5.2 \ {\rm km \ s^{-1}}$
q	$1.00\pm0.01$
d	$11.1 \pm 1.7 \; \mathrm{kpc}$

Table 3.12 Best particle swarm fit of kinematic and potential parameters of a single simulated stream orbit data without distance perturbation and with imperfect rotation curve.

Parameter	Value
R	$10.000 \pm 0.001 \; {\rm kpc}$
$V_X$	$-100.0 \pm 0.1 \text{ km s}^{-1}$
$v_y$	$-300.0 \pm 0.1 \text{ km s}^{-1}$
$V_Z$	$-100.0 \pm 0.1 \text{ km s}^{-1}$
V <sub>halo</sub>	$115.00 \pm 0.02 \text{ km s}^{-1}$
q	$1.00\pm0.01$
d	$12.00\pm0.01\;\mathrm{kpc}$

Table 3.13 Best particle swarm fit of kinematic and potential parameters of a single simulated stream orbit data without distance perturbation and with perfect rotation curve.

This shows that the inability to fit the halo structure parameters is not a result of any property of the orbit itself. If the orbit was perfectly known, the potential parameters can be recovered. Therefore, any deviation from the true halo parameters is a manifestation of three effects: the distance perturbation arising from the absolute magnitude distribution, the rotation curve perturbation arising from anticipated observational error, and the inherent dispersion of the stream quantities. It is therefore concluded that the dispersion of the stream is responsible for the bad fits of q, and this single stream cannot recover the halo structure parameters to the required accuracy.

#### 3.2.2.3 Constraining Halo Mass in Incorrect Halo

This section will briefly address the issue of constraining the halo mass of a stream when it is fit in an incorrect potential.

Utilizing the same stream as the previous two sections, which was generated

in a spherical logarithmic halo, we will fit its kinematic parameters and simulated rotation curve within a spherical NFW halo with variable  $v_{c,max}$  and  $r_s$ . The logarithmic and NFW halo models are not particularly different, and thus we expect to be able to recover the total halo mass. To avoid local minima, we fit via particle swarm with N = 100 particles. The best fit solution, with errors, is shown in Table 3.14. The  $\chi^2$  for this fit is 3.74.

Parameter	Value
R	$9.7\pm0.6~{ m kpc}$
$V_x$	$-93\pm4~\mathrm{km~s^{-1}}$
$V_y$	$-290 \pm 9 {\rm ~km~s^{-1}}$
$V_{Z}$	$-101 \pm 3 {\rm ~km~s^{-1}}$
V <sub>c,max</sub>	$144 \pm 37 \ {\rm km \ s^{-1}}$
rs	$49.8\pm3.4~\rm kpc$
$\chi^2$	3.74

Table 3.14 Best particle swarm fit of kinematic and potential parameters of a single simulated stream within NFW Galactic potential.

To understand how the halo masses compare, we must utilize the potentials from the previous chapter and Poisson's equation to obtain the mass density as a function of radius. We then integrate this density to a standard distance to calculate the enclosed mass. Figure 3.3 shows the simulated stream being enclosed within  $R_{GC} < 60$  kpc. This will be the distance used for the subsequent mass calculations.

Let  $r^2 = X_{GC}^2 + Y_{GC}^2 + Z_{GC}^2.$  The density of the logarithmic halo given in Equation 1.4 is

$$\rho_{\text{Log}}(r) = \frac{1}{4\pi r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi_{\text{Log}}}{\partial r} \right)$$
$$= \frac{v_{halo}^2}{2\pi} \left( \frac{3}{r^2 + d^2} - \frac{2r^2}{(r^2 + d^2)^2} \right)$$

The mass enclosed within 60 kpc for the logarithmic halo is

$$\begin{split} \mathcal{M}_{\text{Log}}(\mathbf{r} < 60 \text{ kpc}) &= \int_{0}^{60 \text{ kpc}} 4\pi r^{2} \rho_{Log}(r) dr \\ &= 3.3 \times 10^{11} \text{ M}_{\text{Sun}}. \end{split}$$

For comparison, the density and enclosed mass within the best-fit NFW halo are

$$\rho_{\rm NFW}(r) = \frac{1}{4\pi r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi_{\rm Log}}{\partial r} \right)$$
$$= -\frac{r_{\rm s} v_{\rm c,max}^2}{4\pi r^2 \times 0.216} \left( \frac{r_s}{(r+r_s)^2} - \frac{1}{1+\frac{r_s}{r_s}} \right)$$

and

$$\begin{split} M_{\rm NFW}({\rm r} < 60 \ {\rm kpc}) &= \int_0^{60 \ {\rm kpc}} 4\pi r^2 \rho_{\rm NFW}(r) dr \\ &= 2.6 \times 10^{11} \ {\rm M}_{\rm Sun}. \end{split}$$

Therefore, the NFW halo mass is broadly consistent with the logarithmic halo mass. This is an important result. If we assume a spherical halo and fit a tidal stream created in one halo within the model of another halo, we find the halo mass to be broadly consistent. Therefore, while a single stream may not place tight constraints on a halo's structure parameters, if a spherical halo is assumed, it is capable of broadly recovering the halo mass.

#### 3.3 Fitting Two Tidal Streams

The previous section demonstrated that fitting orbits to a single well-defined tidal stream is possible, and can reliably retrieve the kinematic parameters that created the stream. However, the halo structure parameters were not recovered to the desired accuracy. This may be due to the fact that one stream only samples a small part of the Galactic halo. While its orbit may take it out to large distances and other areas of the sky, the region sampled by one stream is relatively small. Fitting two streams will allow us to sample different areas of the sky, and may place more constraints on the halo.

#### 3.3.1 Two Simulated Streams: Unknown Logarithmic Halo

We now endeavour to fit two tidal streams within the Galactic potential. The first stream will be the same one used in the previous section, except evolved in a logarithmic halo with  $v_{halo} = 115$  km s<sup>-1</sup>, q = 1.5, and d = 15 kpc. The purpose of these differences is to see whether a two stream fit will converge on a nonspherical halo. The new stream will have kinematic parameters  $(l, b, R, v_x, v_y, v_z) =$  $(220^\circ, 53.5^\circ, 30 \text{ kpc}, -200 \text{ km s}^{-1}, 100 \text{ km s}^{-1}, 100 \text{ km s}^{-1})$ . It will be made to represent a slightly larger dwarf galaxy, and thus modeled by a Plummer sphere with a = 0.3 kpc and  $M_P = 15$  M<sub>u</sub>. We are required to regenerate the detections of Stream 1 from the previous section since the halo parameters have been changed. The simulated detections for this Stream 1 are given in Table 3.15, those of Stream 2 are given in Table 3.16 and the streams are shown in Figure 3.5. We use the  $l = 225^{\circ}$  and  $l = 275^{\circ}$  points to obtain a velocity guess for Stream 2 of v = $(-165 \text{ km s}^{-1}, 50 \text{ km s}^{-1}, 105 \text{ km s}^{-1})$ . We use the same velocity guess for Stream 1 as in the previous section.

l (°)	$b(^{\circ})$	$\delta b$ (°)	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} (\mathrm{km} \mathrm{s}^{-1})$	$d_{Sun}\;(kpc)$	$\delta d_{Sun} (kpc)$
75	29.43	0.10	-221.5	0.3	24.8	0.4
100	46.33	0.16	-209.3	0.5	14.7	0.6
125	54.60	0.16	-158.7	0.4	12.0	0.7
150	56.63	0.15	-104.2	0.4	9.2	0.7
170	54.83	0.16	-55.9	0.7	10.6	0.4
200	44.63	0.22	43.7	0.7	10.7	0.5
225	23.18	0.18	157.8	0.7	11.3	0.4
250	-7.40	0.15	210.1	0.3	21.0	0.3

Table 3.15 Stream 1 dataset for prolate logarithmic halo with  $v_{halo} = 115$  km s<sup>-1</sup>, q = 1.5 and d = 12 kpc.

l (°)	b(°)	$\delta b$ (°)	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} (\mathrm{km} \mathrm{s}^{-1})$	$d_{\rm Sun}~({\rm kpc})$	$\delta d_{Sun} (kpc)$
175	46.25	0.08	97.9	0.8	50.0	0.7
200	51.71	0.08	136.2	0.6	37.1	1.0
220	53.55	0.13	133.1	0.5	30.5	0.8
250	51.11	0.13	101.8	0.2	26.2	1.0
275	42.08	0.16	47.7	0.6	23.1	1.1
300	24.61	0.15	-42.1	1.2	21.6	0.6
325	-1.43	0.14	-135.3	1.1	26.1	0.6
350	-23.13	0.06	-162.4	0.6	39.8	1.1

Table 3.16 Stream 2 dataset for prolate logarithmic halo with  $v_{halo} = 115 \text{ km s}^{-1}$ , q = 1.5 and d = 12 kpc.

We showed previously that the kinematics of one stream can be reliably fit in both known and unknown halos. We will therefore not repeat this exercise with two streams. We will also not repeat fitting in a variable halo without rotation curve data, since we showed this to be fruitless. Instead, we will proceed immediately to fitting two streams in a logarithmic halo with variable  $v_{halo}$ , q and d and a simulated



Figure 3.5 Simulated tidal streams in logarithmic halo with  $v_{halo} = 115$  km s<sup>-1</sup>, q = 1.5 and d = 12 kpc. Top panel shows both streams and true orbits in (l, b) sky coordinates. Middle shows both streams radial velocity  $v_{gsr}$  as a function of l, while bottom shows perturbed distances  $d_{Sun}$  as a function of l.

rotation curve. Finally, we will fit two tidal streams in a triaxial halo and show that all triaxial parameters can be recovered.

Fitting two streams within a variable logarithmic halo translates into minimizing a metric with eleven parameters. Increasing the parameter count from seven to eleven drastically increases the dimensionality of the minimization. We showed in the previous section that particle swarm is always capable of finding the same minima as gradient search, and never finds local minima. As the number of parameters increases, we expect the surface to become more contaminated with local minima. We will therefore not attempt these fits with gradient search, and will rely on particle swarm.

The best fit parameters of a particle swarm optimization are given in Table 3.17. The fitness of this solution is  $\chi^2 = 23.5$ .

We can see that the halo speed and scale length are not well recovered via

Parameter	Value
$R_1$	$9.8\pm0.6~\rm kpc$
$v_{1,x}$	$-96\pm2~{\rm km~s^{-1}}$
$v_{1,y}$	$-294\pm4~{\rm km~s^{-1}}$
$v_{1,z}$	$-99\pm2~{\rm km~s^{-1}}$
$R_2$	$25.8\pm1.4~\rm kpc$
$V_{2,x}$	$-196 \pm 5 \text{ km s}^{-1}$
$V_{2,y}$	$100\pm5~{\rm km~s^{-1}}$
$V_{2,z}$	$102 \pm 2 \text{ km s}^{-1}$
$v_{halo}$	$145 \pm 11 {\rm ~km~s^{-1}}$
q	$1.52\pm0.11$
d	$24.1\pm3.0\;\mathrm{kpc}$
$\chi^2$	23.5

Table 3.17 Best particle swarm fit of kinematic and potential parameters of two simulated streams.

these two streams, and are bothered by a strange peculiarity in the distance  $R_2$ . The true value is  $R_2 = 30.0$  kpc while the fit value is  $R_2 = 25.8$  kpc. The velocities being consistent, it is odd that the particle swarm would find  $R_2$  to be so blatently inconsistent with the true value. If the radial velocity errors are expanded to 5 km s<sup>-1</sup>, the correct  $R_2$  and halo values are recovered. On the face, this seems contradictory to the findings of the previous section: smaller velocity errors should recover the parameters better. However, we'd like to draw attention to the  $l = 175^{\circ}$ ,  $l = 325^{\circ}$  and  $l = 350^{\circ}$  points in Figure 3.5. These points, being near the ends of the stream, possess velocities that have small errors, but are not consistent with the orbit value to within 2  $\sigma$  confidence, as can be seen in Figure 3.6. The particle swarm is fitting these velocities, attempting to find an orbit that is consistent with them, but sacrificing accuracy in  $R_2$  and the halo parameters to obtain a good fit. This is also driving up the  $\chi^2$  value. If these points are eliminated from the dataset, the best fit parameters become those shown in Table 3.18, with a fitness value of  $\chi^2 = 3.00$ . The best fit and true orbits are shown, along with the stream detections, in Figure 3.7. We see two valuable insights from this: the distance  $R_2$  and the kinematic parameters are extremely well recovered once we eliminate points from the stream ends. However, the fit to the halo flattening q is sacrificed. This serves as a warning: radial velocities at the end of a stream may be inconsistent with the

true orbit of that stream, and one must be extremely aware when using them in fits. They may help to constrain halo structure parameters, but at the expense of an obvious kinematic best fit.



Figure 3.6 View of Figure 3.5 in range  $170^{\circ} < l < 180^{\circ}$ . Shown in red is the radial velocity of the simulated tidal stream, with radial velocity determined by mean clipping shown in pink. The clipped mean accurately determines the mean radial velocity, but has errors that make it inconsistent with both the true and fit orbits. This is due to the stream deviating from the orbit near the far ends. This is remedied by increasing the velocity errors of the simulated streams to observationally reasonable values.

To remedy this problem, all fits for Streams 1 and 2 subsequent to this will use radial velocity errors that are larger than the clipped mean predictions, but still experimentally plausible. These are reflected in Tables 3.19 and 3.20.

A particle swarm optimization on these new stream datasets gives best fit parameters shown in Table 3.21. The fitness of this solution is  $\chi^2 = 1.18$ . The best fit and true orbits are shown, along with the stream detections, in Figure 3.7.

This section shows that two streams can broadly fit the halo flattening, that is they can determine that the halo is "definitely" prolate. We see, however, that

Value
$10.0\pm0.7\;\mathrm{kpc}$
$-98 \pm 4 \mathrm{~km~s^{-1}}$
$-295 \pm 7 \text{ km s}^{-1}$
$-100 \pm 2 \text{ km s}^{-1}$
$29.9\pm2.8\;\mathrm{kpc}$
$-201\pm9~\rm km~s^{-1}$
$104 \pm 8 \text{ km s}^{-1}$
$101 \pm 3 \mathrm{~km~s^{-1}}$
$132\pm13~\mathrm{km~s^{-1}}$
$1.28\pm0.19$
$20.6\pm4.4~\rm kpc$
3.00

Table 3.18 Best particle swarm fit of kinematic and potential parameters of two simulated streams without endpoints.

l (°)	b(°)	$\delta b~(^\circ)$	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} (\mathrm{km} \mathrm{s}^{-1})$	$d_{Sun} (kpc)$	$\delta d_{Sun} (kpc)$
75	29.43	0.25	-221	5	24.8	0.4
100	46.33	0.25	-209	5	14.7	0.6
125	54.60	0.25	-159	5	12.0	0.7
150	56.63	0.25	-104	5	9.2	0.7
170	54.83	0.25	-56	5	10.6	0.4
200	44.63	0.25	44	5	10.7	0.5
225	23.18	0.25	159	5	11.3	0.4
250	-7.40	0.25	210	5	21.0	0.3

Table 3.19 Stream 1 dataset for prolate logarithmic halo with  $v_{halo} = 115 \text{ km s}^{-1}$ , q = 1.5 and d = 12 kpc with expanded velocity errors.

the halo scale length is not consistent with the true value to within  $2\sigma$  confidence, and depending on the velocities chosen on the stream, the halo flattening fits may be unreliable.

To test the effect of a particular rotation curve perturbation, we perform four additional particle swarm optimizations with different rotation curves. The values of d are given, with errors, in Table 3.22. We can see that the halo scale length is not consistent with the true value.

l (°)	b(°)	$\delta b$ (°)	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} (\mathrm{km \ s^{-1}})$	$d_{Sun} (kpc)$	$\delta d_{Sun} (kpc)$
175	46.25	1.0	97	10	50.0	0.7
200	51.70	1.0	136	5	37.1	1.0
225	53.55	1.0	133	5	30.5	0.8
250	51.11	1.0	102	5	26.2	1.0
275	42.08	1.0	48	5	23.1	1.1
300	24.61	1.0	-42	10	21.6	0.6
325	-1.43	1.0	-135	10	26.2	0.6
350	-23.13	0.5	-162	5	39.8	1.1

Table 3.20 Stream 2 dataset for prolate logarithmic halo with  $v_{halo} = 115 \text{ km s}^{-1}$ , q = 1.5 and d = 12 kpc with expanded velocity errors.

Parameter	Value
$R_1$	$9.7\pm0.8~{\rm kpc}$
$v_{1,x}$	$-93\pm8~{\rm km~s^{-1}}$
$v_{1,y}$	$-289 \pm 15 \text{ km s}^{-1}$
$v_{1,z}$	$-101 \pm 10 \text{ km s}^{-1}$
$R_2$	$30.6\pm1.9\;\mathrm{kpc}$
$v_{2,x}$	$-194 \pm 15 \; \rm km \; s^{-1}$
$V_{2,y}$	$92\pm12~{\rm km~s^{-1}}$
$v_{2,z}$	$104\pm14~\rm km~s^{-1}$
V <sub>halo</sub>	$117 \pm 15 \ {\rm km \ s^{-1}}$
q	$1.61\pm0.38$
d	$19.7\pm2.0~{ m kpc}$
$\chi^2$	1.18

Table 3.21 Best particle swarm fit of kinematic and potential parameters of two simulated streams with experimentally plausible velocity errors.

#### 3.3.2 Two Simulated Streams: Unknown Triaxial Halo

The previous section demonstrated that two streams can do what one cannot: two of the three logarithmic halo parameters  $(v_{halo} \text{ and } q)$  can be fit reliably when using two streams and a simulated rotation curve. The fit halo scale length d is not consistent with the true value.

Despite this limitation, we are curious how far we can push this ability. We therefore move to fitting two streams in a triaxial halo with all parameters unknown:  $v_{halo,t}, q_1, q_z, d_t$ , and  $\phi$ . Both streams were regenerated in a triaxial halo with the parameters found by Law and Majewski (2010):  $(v_{halo,t}, q_1, q_z, d_t, \text{ and } \phi) = (115 \text{ km s}^{-1}, 1.38, 1.36, 15 \text{ kpc}, 97^{\circ})$ . We use these parameters in particular because



Figure 3.7 Top panel shows true and fit orbits in (l, b) coordinates, middle panel shows  $(l, v_{gsr})$ , and bottom panel  $(l, d_{Sun})$ . It can be seen that the fit orbits are good approximations of the true orbits. The fit orbits succeed in correlating with the stream detections. The true orbit deviates from the stream detections at the extreme ends of the streams.

d (kpc)
$19.7\pm2.7$
$21.0\pm2.5$
$18.9\pm4.5$
$19.4\pm3.1$

Table 3.22 Results of four particle swarm optimizations of two simulated streams with potential parameters and different rotation curve perturbations.

LM10 found these to be the best fit for the Sagittarius Dwarf tidal stream, and if they are the best fit for the true Galactic halo, we would like to determine if two simulated streams can find a halo of this shape. We use the same stream kinematics as in the previous subsections. The detections for the two triaxial streams are given in Tables 3.23 and 3.24. The streams are shown in Figure 3.8.

A particle swarm optimization was performed, and the best fit parameters are

l (°)	b(°)	$\delta b~(^\circ)$	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} ({\rm km~s^{-1}})$	$d_{Sun} (kpc)$	$\delta d_{Sun} (kpc)$
75	27.90	0.25	-231	5	27.3	0.4
100	44.46	0.25	-213	5	16.1	0.5
125	53.98	0.25	-160	5	13.4	1.0
150	56.43	0.25	-104	5	10.6	0.8
171	55.21	0.25	-56	5	11.0	0.9
200	45.30	0.25	42	5	10.1	0.4
225	24.88	0.25	154	5	11.2	0.4
250	-4.82	0.25	224	5	20.6	0.4

Table 3.23 Stream 1 dataset for triaxial logarithmic halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ , and  $\phi = 97^{\circ}$ .

l (°)	b(°)	$\delta b~(^\circ)$	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} (\mathrm{km} \mathrm{s}^{-1})$	$d_{Sun}\;(kpc)$	$\delta d_{Sun} \ (kpc)$
150	34.81	0.5	41	5	60.2	3.0
175	44.62	0.25	112	5	48.4	1.1
200	51.59	0.25	138	5	34.5	0.8
220	53.52	0.25	133	5	32.2	0.9
250	50.71	0.25	101	5	27.2	0.8
275	41.72	0.25	45	5	23.9	0.5
300	22.21	0.25	-57	5	23.7	0.7
325	-6.82	0.25	-151	5	32.8	1.3
350	-30.09	0.25	-150	5	45.7	1.6

Table 3.24 Stream 2 dataset for triaxial logarithmic halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ , and  $\phi = 97^{\circ}$ .

given in Table 3.25. The fitness of this solution is  $\chi^2 = 1.60$ . The best fit and true orbits are shown, along with the stream detections, in Figure 3.9. We can see that overall the fit to the halo flattenings  $q_1,q_z$  and the angle  $\phi$  are consistent to within errors. The halo scale length d, however, is not.

#### 3.3.3 Two Simulated Streams: Unknown Spherical + Triaxial Halo

The final case we will consider is that of fitting two tidal streams within a halo that is composed both of triaxial and spherical components. The motivation behind this is the possibility that the Galactic halo may be triaxial near the center of the Galaxy, but become spherical at large distances from the Galactic center. We will therefore consider the case of a gravitational potential that is the sum of a triaxial and spherical logarithmic potentials. The parameters for the tri-



Figure 3.8 Simulated tidal streams in triaxial logarithmic halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ , and  $\phi = 97^\circ$ . Top panel shows both streams and true orbits in (l, b) sky coordinates. Middle shows both streams radial velocity  $v_{gsr}$  as a function of l, while bottom shows perturbed distances  $d_{Sun}$  as a function of l.

axial halo are the same as the previous subsection:  $(v_{halo,t}, q_1, q_z, d_t, \text{ and } \phi) = (115 \text{ km s}^{-1}, 1.38, 1.36, 12 \text{ kpc}, 97^{\circ})$ . The large spherical halo will have parameters  $(v_{halo}, d) = (200 \text{ km s}^{-1}, 50 \text{ kpc})$ .

A particle swarm optimization was conducted, the best fit parameters being given in Table 3.28. The fitness of this solution is  $\chi^2 = 1.73$ . The best fit and true orbits are shown, along with the stream detections, in Figure 3.11. We can see that these two streams are incapable of fitting the scale length and mass of the external spherical halo to within errors. We will pursue fitting this halo in the next section with three streams.

#### 3.3.4 Discussion

The findings of this section are enumerated as follows:

Parameter	Value		
$R_1$	$10.2\pm0.7\;{\rm kpc}$		
$v_{1,x}$	$-90\pm7~{\rm km~s^{-1}}$		
$v_{1,y}$	$-299 \pm 11 \text{ km s}^{-1}$		
$v_{1,z}$	$-101 \pm 8 \text{ km s}^{-1}$		
$R_2$	$30.8 \pm 2.2 \; \mathrm{kpc}$		
$v_{2,x}$	$-198 \pm 13 \; \rm km \; \rm s^{-1}$		
$v_{2,y}$	$103 \pm 12 \text{ km s}^{-1}$		
$V_{2,z}$	$102 \pm 11 {\rm ~km~s^{-1}}$		
$v_{halo}$	$121 \pm 10 {\rm ~km~s^{-1}}$		
$q_1$	$1.25\pm0.12$		
$q_z$	$1.29\pm0.10$		
d	$16.6\pm1.7\;\mathrm{kpc}$		
$\phi$	$96^{\circ} \pm 10^{\circ}$		
$\chi^2$	1.60		
$\begin{array}{c} \mathrm{V}_{1,\mathrm{y}} \\ \mathrm{V}_{1,\mathrm{z}} \\ \mathrm{R}_{2} \\ \mathrm{V}_{2,\mathrm{x}} \\ \mathrm{V}_{2,\mathrm{y}} \\ \mathrm{V}_{2,\mathrm{z}} \\ \mathrm{V}_{\mathrm{halo}} \\ \mathrm{q}_{1} \\ \mathrm{q}_{\mathrm{z}} \\ \mathrm{d} \\ \phi \\ \chi^{2} \end{array}$	$-101 \pm 8 \text{ km s}^{-1}$ $30.8 \pm 2.2 \text{ kpc}$ $-198 \pm 13 \text{ km s}^{-1}$ $103 \pm 12 \text{ km s}^{-1}$ $102 \pm 11 \text{ km s}^{-1}$ $121 \pm 10 \text{ km s}^{-1}$ $1.25 \pm 0.12$ $1.29 \pm 0.10$ $16.6 \pm 1.7 \text{ kpc}$ $96^{\circ} \pm 10^{\circ}$ $1.60$		

Table 3.25 Best particle swarm fit of kinematic and potential parameters of two simulated streams in triaxial halo.

l (°)	b(°)	$\delta b$ (°)	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} (\mathrm{km \ s^{-1}})$	$d_{\rm Sun}~({\rm kpc})$	$\delta d_{Sun} (kpc)$
75	31.28	0.25	-228	5	20.8	0.3
100	46.96	0.25	-200	5	13.0	0.4
125	54.86	0.25	-152	5	11.2	0.5
150	56.70	0.25	-102	5	10.3	0.6
170	55.12	0.25	-56	5	11.1	0.6
200	44.94	0.25	34	5	10.5	0.5
225	25.26	0.25	135	5	11.1	0.3
250	-2.08	0.25	204	5	16.6	0.3

Table 3.26 Stream 1 dataset for triaxial+spherical logarithmic halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ ,  $\phi = 97^{\circ}$ ,  $v_{halo} = 200 \text{ km s}^{-1}$  and d = 50 kpc.

- Two tidal streams are capable of doing what one cannot. Halo parameters such as the Z-direction flattening are well fit in axisymmetric halos. Flattenings and halo orientation angles are well fit in triaxial halos. Both of these are provided that the radial velocity errors are large enough to allow the orbit fit to seek an overall global minimum.
- Halo scale lengths are not well fit by the two streams analyzed above.
- The combination triaxial+spherical halo is not well recovered by two streams.


Figure 3.9 True and best fit orbits in triaxial halo. Top panel shows (l, b) coordinates, middle panel shows  $(l, v_{gsr})$ , and bottom panel  $(l, d_{Sun})$ .

l (°)	b(°)	$\delta b$ (°)	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} (\mathrm{km \ s^{-1}})$	$d_{\rm Sun}~({\rm kpc})$	$\delta d_{Sun} (kpc)$
175	45.55	0.25	42	5	38.2	1.0
200	51.45	0.25	112	5	34.5	0.6
220	53.62	0.25	135	5	30.7	0.8
250	51.25	0.25	126	5	27.3	0.6
275	43.05	0.25	82	5	23.7	0.6
300	25.93	0.25	-13	5	24.2	0.8

Table 3.27 Stream 2 dataset for triaxial+spherical logarithmic halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ ,  $\phi = 97^{\circ}$ ,  $v_{halo} = 200 \text{ km s}^{-1}$  and d = 50 kpc.

# 3.4 Three Stream Fitting: Including a Sagittarius Dwarf Emulator

We now move to fitting three tidal streams within a simulated Galactic potential. The previous section showed that flattening and orientation quantities are well recovered by the two streams previously analyzed. However, any model of the Galaxy must include the Sagittarius Dwarf Tidal Stream (Sgr). Since Sgr spans the

Parameter	Value
$R_1$	$10.0\pm0.6\;\mathrm{kpc}$
$v_{1,x}$	$-107 \pm 9 \text{ km s}^{-1}$
$v_{1,y}$	$-324 \pm 11 \text{ km s}^{-1}$
$v_{1,z}$	$-107 \pm 8 {\rm ~km~s^{-1}}$
$R_2$	$30.9\pm1.2\;{\rm kpc}$
$v_{2,x}$	$-224 \pm 15 \text{ km s}^{-1}$
$v_{2,y}$	$131 \pm 14 \text{ km s}^{-1}$
$v_{2,z}$	$99 \pm 12 \text{ km s}^{-1}$
$v_{halo,t}$	$128 \pm 17 \ {\rm km \ s^{-1}}$
$q_1$	$1.27\pm0.15$
$q_z$	$1.39\pm0.16$
$\rm d_{halo,t}$	$10.6\pm1.6\;\mathrm{kpc}$
$\phi$	$100^{\circ} \pm 18^{\circ}$
V <sub>halo</sub>	$171 \pm 20 \text{ km s}^{-1}$
$\mathrm{d}_{\mathrm{halo}}$	$21.5\pm1.6\;\mathrm{kpc}$
$\chi^2$	1.73

Table 3.28 Best particle swarm fit of kinematic and potential parameters of two simulated streams in triaxial+spherical halo.

entire sky and has a well-known progenitor. We endeavour to see if including an emulator of this stream will provide constraints on the halo scale length.

#### 3.4.1 Generation of a Test Sgr Stream

The first model of the Sgr tidal stream that satisfies all sky position, radial velocity, and distance constraints was developed by Law and Majewski (2010, LM) by fitting a dwarf galaxy evolving in a triaxial Galactic halo. Prior to this, Law, Johnston and Majewski (2005, LJM) published a near-consistent model using the axisymmetric logarithmic halo.

These two models differ slightly in their initial kinematics. For the purposes of this section, though, we simply wish to see if a simulated dwarf galaxy stream can be fit, and have its kinematics recovered. We therefore select kinematics consistent with the LJM 2005 model:  $(l, b, R, v_x, v_y, v_z) = (5.6^\circ, -14.2^\circ, 28 \text{ kpc},$ 238 km s<sup>-1</sup>, -42 km s<sup>-1</sup>, 222 km s<sup>-1</sup>). Dwarf parameters of  $M_{Sgr} = 1 \times 10^9 \text{ M}_{sun} \approx$ 4498.66 M<sub>u</sub>. With a dwarf galaxy, the effects of energy segregation described in the previous chapter become more pronounced. We therefore include distance scale



Figure 3.10 Simulated tidal streams in triaxial+spherical halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ ,  $\phi = 97^{\circ}$ ,  $v_{halo} = 200 \text{ km s}^{-1}$  and d = 50 kpc. Top panel shows both streams and true orbits in (l, b) sky coordinates. Middle shows both streams radial velocity  $v_{gsr}$  as a function of l, while bottom shows perturbed distances  $d_{Sun}$  as a function of l.

factors in stream fitting. We will also present the simulated Sgr stream in the Sagittarius Dwarf coordinate system ( $\theta = \Lambda_{\text{Sgr,GC}}, \phi = B_{\text{Sgr,GC}}$ ).

As LJM and LM have concluded that an axisymmetric halo is insufficient to represent the Galactic halo mass distribution, we will not consider three stream fitting in this case. Instead, we will consider the triaxial and triaxial+spherical cases given in the previous section.

The simulated Sgr stream in the triaxial halo is shown in Figure 3.12, with detections given in Table 3.29. The simulated Sgr stream in the triaxial+spherical halo is shown in Figure 3.13, with detections given in Table 3.30.

A particle swarm optimization was performed, with best fit parameters given in Table 3.31. The fitness of this solution is  $\chi^2 = 4.67$ , and scale factors of  $S_{\text{Leading}} =$ 1.00 and  $S_{\text{Trailing}} = 0.59$  were fit. The best fit and true orbits for Streams 1 and 2



Figure 3.11 Top panel shows true and fit orbits in (l, b) coordinates, middle panel shows  $(l, v_{gsr})$ , and bottom panel  $(l, d_{Sun})$  for triaxial+spherical halo with  $v_{halo,t} =$ 115 km s<sup>-1</sup>,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12$  kpc,  $\phi = 97^{\circ}$ ,  $v_{halo} = 200$  km s<sup>-1</sup> and d = 50 kpc.

are shown, along with the stream detections, in Figure 3.14. The simulated Sgr fit orbit with detections is shown in Figure 3.15.

For the case of the triaxial+spherical halo, a particle swarm optimization was performed, with best fit parameters shown in Table 3.32. The fitness of this solution is  $\chi^2 = 8.11$ , and scale factors of  $S_{\text{Leading}} = 1.00$  and  $S_{\text{Trailing}} = 0.76$  were fit. The best fit orbits are shown in Figures 3.16 and 3.17 and It is clear that the three streams modeled here cannot successfully reproduce the parameters of a combination triaxial+spherical halo.

As a final case before moving on to real streams, we fit the streams generated in the triaxial + spherical halo with orbits in a triaxial halo. Suppose the "true" Galactic halo was a combination of triaxial and spherical. The aim of this test is to see whether a simple triaxial fit would converge in this case. We utilize the same streams created in the triaxial+spherical case above, and a particle swarm

bgr,GC	$() = \operatorname{Sgr}, \operatorname{GC}()$	)**()	* ysi ( »	) • • ysi ( »	)		~)
275	-0.92	4.26	14	3	76.4	3.2	L
290	-0.12	3.47	66	2	76.3	2.4	L
300	-4.88	7.02	110	27	72.5	2.5	L
325	-2.06	4.04	163	4	44.9	2.6	$\mathbf{L}$
350	-1.45	4.11	180	1	32.9	0.4	$\mathbf{L}$
5	-1.59	1.90	177	4	33.2	1.6	Т
25	-2.21	4.64	127	10	36.3	9.2	Т
50	-2.40	5.00	17	7	28.9	5.3	Т
75	-2.20	19.92	-78	4	27.7	3.4	Т
100	-0.26	16.71	-139	5	42.1	3.1	Т
125	1.71	9.62	-144	4	64.4	2.0	Т
150	1.67	8.45	-71	4	102.9	3.0	Т
165	3.52	27.51	8	11	118.8	5.6	Т

 $\Lambda_{\text{Sgr,GC}}$  (°)  $B_{\text{Sgr,GC}}$  (°)  $\delta b$  (°)  $v_{asr}$  (km s<sup>-1</sup>)  $\delta v_{asr}$  (km s<sup>-1</sup>)  $d_{\text{Sun}}$  (kpc)  $\delta d_{\text{Sun}}$  (kpc)

Table 3.29 Sgr dataset for triaxial logarithmic halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ ,  $\phi = 97^{\circ}$ . Leading and trailing stream segments are denoted L and T, respectively.

optimization gives a best fit  $\chi^2$  of 84.3. We therefore conclude that if a stream is evolving within a triaxial+spherical halo, we are unable to fit it with a simple triaxial model.

## 3.5 Discussion

This completes our study of fitting orbits to simulated tidal streams chosen to mimic the GD-1, Orphan, and Sagittarius streams. The effective conclusions of this chapter are given as follows.

- Stream kinematics for our simulated streams can be constrained in spherical halos, with or without rotation curve fitting.
- Adding halo rotation curves to stream fits aids in finding the best fit halo parameters.
- The total mass of a spherical halo is well constrained, even when the halo is a different model than that used to create the stream.
- Fits of the flattening q suffer systematic biases. We have shown these biases to be caused by the dispersion in stream quantities, and if the stream mirrored

	$- \operatorname{Sgl}, \operatorname{GC}()$		~ ysi ( ~	) * * ysi ( %	) "Bull (P *) *	-Sun (P	~)
25	-2.39	0.97	114	11	27.9	2.3	Т
50	-0.50	0.85	51	3	19.5	0.7	Т
100	1.13	0.99	-118	3	29.3	1.5	Т
125	1.12	0.63	-124	3	32.0	0.9	Т
150	2.66	0.38	-51	3	45.2	1.3	Т
175	3.42	0.37	74	4	42.9	1.3	Т
200	5.55	0.41	148	4	34.5	2.1	Т
100	-5.14	3.10	178	6	18.4	2.6	$\mathbf{L}$
150	-7.18	2.49	-12	18	12.7	3.8	L
200	-8.11	2.01	-147	4	14.4	0.7	$\mathbf{L}$
250	-3.70	0.91	-159	4	25.7	1.1	$\mathbf{L}$
300	-0.83	0.52	-1	2	45.5	1.1	$\mathbf{L}$
350	-1.32	0.42	167	3	33.8	1.4	$\mathbf{L}$

 $\Lambda_{\text{Sgr,GC}}$  (°)  $B_{\text{Sgr,GC}}$  (°)  $\delta b$  (°)  $v_{qsr}$  (km s<sup>-1</sup>)  $\delta v_{qsr}$  (km s<sup>-1</sup>)  $d_{\text{Sun}}$  (kpc)  $\delta d_{\text{Sun}}$  (kpc)

Table 3.30 Sgr dataset for triaxial+spherical logarithmic halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ ,  $\phi = 97^{\circ}$ ,  $v_{halo} = 200 \text{ km s}^{-1}$  and d = 50 kpc.

its orbit perfectly, the halo flattening can be recovered.

- Our clipped mean technique, while accurately determining stream quantities and errors, causes the optimization algorithm to converge to poor minima. This is especially apparent at the ends of streams, where the radial velocity of the stream can deviate from the orbit's prediction. Expanding radial velocity and angle errors to experimentally appropriate levels alleviates this difficulty, and leads to good fits.
- Gradient search is an effective method for finding best fit kinematics of a single stream in a known potential, but quickly collapses when more streams and parameters are introduced. We have therefore adopted particle swarm optimization as the method of choice for multiple streams.
- Two streams chosen to mimic the GD-1 and Orphan Streams in a triaxial halo are able to constrain the structure parameters (mass and flattenings), but are unable to recover the scale length. The constraints on the scale length are obtained by introducing a Sgr emulator stream.



Figure 3.12 Simulated Sgr tidal stream in triaxial halo. Top panel shows true and fit orbits in ( $\Lambda_{\text{Sgr,GC}}, B_{\text{Sgr,GC}}$ ) coordinates, middle panel shows ( $\Lambda_{\text{Sgr,GC}}, v_{gsr}$ ), and bottom panel ( $\Lambda_{\text{Sgr,GC}}, d_{Sun}$ ) for triaxial halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ ,  $\phi = 97^{\circ}$ .

- We are unable to recover parameters from a triaxial+spherical halo with three streams.
- Given a true halo that is triaxial+spherical, stream parameters are not recovered when fit with a triaxial model. This provides two a crucial insights: we are able to determine that a triaxial halo is not a good fit to streams generated in a triaxial+spherical halo, and conversely, if true streams are not fit well in a triaxial halo, an additional halo component may be responsible for the discrepancy.



Figure 3.13 Simulated Sgr tidal stream in triaxial+spherical halo. Top panel shows true and fit orbits in ( $\Lambda_{\text{Sgr,GC}}, B_{\text{Sgr,GC}}$ ) coordinates, middle panel shows ( $\Lambda_{\text{Sgr,GC}}, v_{gsr}$ ), and bottom panel ( $\Lambda_{\text{Sgr,GC}}, d_{Sun}$ ) for triaxial+spherical halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ ,  $\phi = 97^{\circ}$ ,  $v_{halo} = 200 \text{ km s}^{-1}$  and d = 50 kpc.

Parameter	Value
$R_1$	$10.5\pm0.7~\rm kpc$
$v_{1,x}$	$-94\pm7~{\rm km~s^{-1}}$
$v_{1,y}$	$-310 \pm 9 {\rm ~km~s^{-1}}$
$v_{1,z}$	$-104 \pm 8 \text{ km s}^{-1}$
$R_2$	$31.1 \pm 2.1 \text{ kpc}$
$V_{2,x}$	$-200 \pm 13 \text{ km s}^{-1}$
$v_{2,y}$	$107 \pm 11 {\rm ~km~s^{-1}}$
$V_{2,z}$	$103 \pm 12 \text{ km s}^{-1}$
$R_3$	$28.8\pm3.9\;{\rm kpc}$
$v_{3,x}$	$238 \pm 19 \ {\rm km \ s^{-1}}$
$v_{3,y}$	$-30 \pm 10 \text{ km s}^{-1}$
$v_{3,z}$	$231\pm6~{\rm km~s^{-1}}$
$v_{halo,t}$	$121\pm 6~{\rm km~s^{-1}}$
$q_1$	$1.29\pm0.09$
$q_z$	$1.25\pm0.06$
$d_{\rm halo,t}$	$11.7\pm0.9\;{\rm kpc}$
$\phi$	$101^{\circ}\pm8^{\circ}$
$\chi^2$	4.67

Table 3.31 Best particle swarm fit of kinematic and potential parameters of three simulated streams in triaxial halo.



Figure 3.14 Best fit and true orbits for Streams 1 and 2 in triaxial halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ ,  $\phi = 97^{\circ}$ .



Figure 3.15 Best fit and true orbits for Stream 3 in triaxial halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ ,  $\phi = 97^{\circ}$ . Overlayed is a 10,000 particle N-body run with the fit parameters evolved over 4 Gyr. We see good agreement in sky position and radial velocity, and consistent agreement in stream distance. Fitting the orbits with distance scale factors is a good first-order approximation to fitting the actual N-body stream distances.

Parameter	Value
$R_1$	$9.7 \pm 1.0 \; \mathrm{kpc}$
$v_{1,x}$	$-101 \pm 21 \text{ km s}^{-1}$
$v_{1,y}$	$-314 \pm 8 \text{ km s}^{-1}$
$v_{1,z}$	$-104 \pm 22 \text{ km s}^{-1}$
$R_2$	$30.8\pm1.7~\rm kpc$
$V_{2,x}$	$-206 \pm 11 \text{ km s}^{-1}$
$v_{2,y}$	$115\pm9~{\rm km~s^{-1}}$
$V_{2,z}$	$97\pm10~{\rm km~s^{-1}}$
$R_3$	$26.0\pm0.7~\rm kpc$
$v_{3,x}$	$254\pm9~{\rm km~s^{-1}}$
$v_{3,y}$	$-41 \pm 9 \text{ km s}^{-1}$
$V_{3,z}$	$240\pm7~{\rm km~s^{-1}}$
$v_{halo,t}$	$132 \pm 13 \text{ km s}^{-1}$
$\mathbf{q}_1$	$1.11\pm0.09$
$q_z$	$1.32\pm0.08$
$d_{\rm halo,t}$	$7.9 \pm 2.3 \; \mathrm{kpc}$
$\phi$	$81^{\circ} \pm 10^{\circ}$
$v_{halo}$	$139 \pm 24 \text{ km s}^{-1}$
$d_{\rm halo}$	$33.5\pm7.6~{\rm kpc}$
$\chi^2$	8.11

Table 3.32 Best particle swarm fit of kinematic and potential parameters of three simulated streams in triaxial+spherical halo.



Figure 3.16 Best fit and true orbits for Streams 1 and 2 in triaxial+spherical halo with  $v_{halo,t} = 115$  km s<sup>-1</sup>,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12$  kpc,  $\phi = 97^{\circ}$ ,  $v_{halo} = 200$  km s<sup>-1</sup> and d = 50 kpc.



Figure 3.17 Best fit and true orbits for Stream 3 in triaxial+spherical halo with  $v_{halo,t} = 115 \text{ km s}^{-1}$ ,  $q_1 = 1.38$ ,  $q_z = 1.36$ ,  $d_t = 12 \text{ kpc}$ ,  $\phi = 97^{\circ}$ ,  $v_{halo} = 200 \text{ km s}^{-1}$  and d = 50 kpc. Overlayed is a 10000 particle N-body run with the fit parameters evolved over 4 Gyr. We see good agreement in sky coordinates and radial velocities, but poor agreement in distances.

# CHAPTER 4 ORBIT FITTING OF GALACTIC TIDAL STREAMS

The previous chapter demonstrated the robustness of the orbit fitting method introduced in Chapter 2. We now apply this method to actual tidal streams orbiting the Galaxy. Before a full simultaneous model can be created for the Galaxy, we will discuss the individual tidal streams to be considered. Models for each of these streams will be developed, after which they will be merged into a full simultaneous orbit fit. The streams that will be considered in this chapter are the Stream of Grillmair and Dionatos (GD-1), the Cetus Polar Stream, and Orphan Stream, and the Sagittarius Dwarf Tidal Stream (Sgr).

### 4.1 The Stream of Grillmair and Dionatos (GD-1)

Grillmair and Dionatos (2006), hereafter GD, announced the detection of a 63° cold stellar stream in the Galactic halo (the stream itself we refer to henceforth as GD-1, following GD), using stellar density counts extracted from the Sloan Digital Sky Survey (SDSS; York et al. 2000). This stream is extremely narrow, less than 0.25° degrees in width, which is less than 50 pc at their measured distances of 7.3 to 9.1 kpc from the Sun. GD therefore concluded that the progenitor was a globular cluster, but the progenitor remains unidentified and could be completely disrupted. Willett et al. (2009) re-extracted the GD-1 stream from SDSS, and supplemented this extraction with radial velocities and metallicities from the Sloan Extension for Galactic Understanding and Exploration (SEGUE, Yanny et al. 2009). This additional kinematic information allowed constraints to be placed on the orbit of the GD-1 stream. The extraction of the stream and the stream's orbital properties will be reiterated here.

Portions of this chapter previously appeared as: Willett, B. A., Newberg, H. J., Zhang, H. T., Yanny, B., Beers, T. C. 2009, "An Orbit Fit for the Grillmair Dionatos Cold Stellar Stream," Astrophysical Journal, 697, p. 207; Newberg, H. J., Yanny, B., Willett, B. A. 2009, "Discovery of a New, Polar-Orbiting Debris Stream in the Milky Way Stellar Halo," Astrophysical Journal, 700L, p. 61N; and Newberg, H. J., Willett, B. A., Yanny, B., Xu, Y. 2010, "The Orbit of the Orphan Stream," Astrophysical Journal, 711, p. 32N.

#### 4.1.1 Photometric Selection

The GD-1 stream was identified in SDSS DR7 by first selecting all stars in the north Galactic cap, and applying color-magnitude filtering techniques using fiducial sequences of known globular clusters. By shifting the filter in magnitude space, we can obtain an estimate of the stream distance, as well as select stream members. Figure 4.1 shows a density map of stars isolated from an M92 fiducial sequence shifted to m - M = 14.76. The stream is shown spanning from RA = 130° at Region 1 to RA = 220° at region 7. Utilizing an F-turnoff magnitude of +4.2 (Cole et al. 2008), the M92 shift suggests a stream distance of 7 to 10 kpc.



Figure 4.1 A Hess diagram of stars in the SDSS footprint which are within the color and magnitude range of the M92 fiducial sequence shifted to m - M = 14.76. The GD-1 stream arcs faintly from  $(\alpha, \delta) = (220^\circ, 58^\circ)$  to  $(126^\circ, 0^\circ)$ . Seven regions where spectroscopy of GD-1 stream star candidates have been obtained are numbered. Squares indicate regions where the stream was clearly found in velocity, and circles indicate additional plates that may contain stream stars.

#### 4.1.2 Photometric Distance estimation

We estimate the distance to the GD-1 stream at the positions of each of the seven regions that it overlaps using a matched filter algorithm. We first generated a Hess diagram from SDSS DR7 data from a region about  $0.5^{\circ}$  wide in RA and  $1^{\circ}$ 

wide in Dec in the vicinity of each plate, centered on the GD-1 stream. Then a Hess diagram of the background was generated from two regions of sky with the same angular extent on the sky, but offset 1.5° higher and 1.5° lower in declination. The background Hess diagram (divided by two to correct for the difference in sky area) was subtracted from the corresponding Hess diagram on the GD-1 stream.

We constructed an M92 filter Hess diagram with the similar method to Grillmair (2009). We first broaden the M92 fiducial sequence from An et al. (2008) with the SDSS photometric errors. Because we do not have a luminosity function for M92 stars, we used the luminosity function of M13, estimated from SDSS survey counts vs. magnitude for stars away from the core of M13, to create the Hess diagram. We assume the distance to M92 is 8.2 kpc. Utilizing the background-subtracted Hess diagrams, the M92 fiducial, and its known distance, we can determine the distance to the stream for every region where a Hess diagram exists. The distance and corresponding error of each of the seven points along the stream with SEGUE spectra are listed in Table 4.1.

We note that the estimated distances to individual regions are in very good agreement with those quoted by Grillmair and Dionatos (2006).

#### 4.1.2.1 Spectroscopy

The SEGUE survey, which is one of three surveys carried out as part of SDSS-II, obtained spectra of approximately 240,000 Milky Way stars toward  $\sim 200$  sightlines that each covered seven square degrees of the sky, with an emphasis on obtaining spectra of fainter halo stars. While most of SEGUE's 200 observing tiles were randomly distributed across the SDSS imaging footprint, a few were placed on streams of known interest, including the GD-1 stream.

All SEGUE spectra were processed through the standard SDSS spectroscopic reduction pipelines (Stoughton et al. 2002), from which radial velocities accurate to about 10 km s<sup>-1</sup> for objects at  $g \sim 19.5$  were determined. In addition, the stellar spectra were processed through the SEGUE stellar parameter pipeline (SSPP; Lee et al. 2008a, 2008b; Allende Prieto et al. 2008) in order to obtain abundance ([Fe/H]), surface gravity (log g), and other atmospheric parameter estimates. We select from the SDSS-II/SEGUE DR7 database all measured parameters of the 12,825 spectra of stars within 1.3° of the GD-1 stream and within the M92 color-magnitude sequence. Most of the 4568 remaining spectra are concentrated in 3° diameter patches centered on SEGUE tiles, but some are part of the SDSS-I and SDSS-II Legacy surveys. These latter surveys targeted nearly the entire SDSS footprint spectroscopically, but with few and limited signal-to-noise on stellar targets (since the SDSS Legacy survey primarily targets galaxy and quasar candidates).



Figure 4.2 The line-of-sight, Galactocentric standard of rest velocity versus Galactic longitude for SEGUE and SDSS stars for which we have spectra, and which are close in color-magnitude space to the fiducial sequence of M92 and close (within 1.3°) in projected distance to the GD-1 stream shown in Figure 4.1. A sine curve with amplitude 110 km s<sup>-1</sup> is shown to indicate the locus of stars rotating with the Sun about the Galactic center. Stars in the halo will have velocities centered on  $v_{gsr} = 0$  and a large  $\sigma \sim 100$  km s<sup>-1</sup> dispersion. Regions where GD-1 stream candidates are followed up on are numbered 1-7. Note in particular the groups of stars at  $v_{gsr} \sim -90$  km s<sup>-1</sup> in regions 5 and 6.

We show in Figure 4.2 the line-of-sight, Galactic standard of rest velocities,  $v_{\rm gsr}$ , for each star in the sample, as a function of Galactic longitude. We calculate

 $v_{\rm gsr}$  using:  $v_{\rm gsr} = \text{RV} + 10.1 \cos b \cos l + 224 \cos b \sin l + 6.7 \sin b$ , where RV is the heliocentric radial velocity in km s<sup>-1</sup> and (l, b) are the standard, Sun-centered Galactic coordinates of each star. A sine curve with amplitude 110 km s<sup>-1</sup> traces an approximate locus of nearby disk stars co-rotating with the Sun. Spheroid stars occupy a broad range of  $v_{gsr}$  centered at  $v_{gsr} = 0$ . Seven regions of interest are marked along the bottom of Figure 4.2, indicating areas with SEGUE plates, where stars identified with the GD-1 stream will be selected. The positions of these seven regions on the sky in equatorial coordinates are also indicated with circles and numbered in Figure 4.1. Regions 5 and 6 were specially targeted by SEGUE with a tile directly on locations along the GD-1 stream.

From examination of Figure 4.2, it appears that there is an excess of stars off the rotating disk locus at  $v_{\rm gsr} \sim -90$  km s<sup>-1</sup> in regions 5 and 6. To confirm that these are in fact GD-1 stream stars, we isolate the stars in regions 5 and 6 and plot their velocity histogram in Figure 4.3.

The distribution in Figure 4.3 is overlayed with Gaussians for the thick disk (dispersion of 30 km s<sup>-1</sup> and an offset of  $\mu \sim 20$  km s<sup>-1</sup>), and inner halo (dispersion of 100 km s<sup>-1</sup>). A significant peak is detected at  $v_{\rm gsr} \sim -82$  km s<sup>-1</sup> which we associate with the GD-1 stream member stars.

We next examine the metallicity distribution of stars in this velocity peak in order to estimate the elemental abundance of the GD-1 stream. Later we will show that the individual  $v_{gsr}$  velocities in regions 5 and 6 are  $71 \pm 2$  and  $87 \pm 2$  km s<sup>-1</sup>, respectively, so we chose a "peak" velocity range of  $-97 < v_{gsr} < -61$  km s<sup>-1</sup>.

Figure 4.4 shows the SSPP abundance estimates for all stars with good metallicity estimates (for a good estimate a turnoff star generally needs to be brighter than about  $g \sim 19$ ). Errors on individual stars [Fe/H] are approximately 0.3 dex for spectral type F objects with g < 18.5. The histogram for all abundances of stars in regions 5 and 6 are plotted with a light line (1311 stars), those for stars in the velocity peak of Figure 4.3 are indicated with a heavy line (115 stars). The stars with velocities of the GD-1 stream are heavily biased towards lower metallicity stars, compared with those of the thick disk ([Fe/H]  $\sim -0.7$ ), or inner halo ([Fe/H]  $\sim -1.6$ ).



Figure 4.3 All stars with spectra from Figure 4.2 towards regions 5, 6 of Figure 4.1 are histogrammed in Galactocentric velocity. Gaussians are overlaid representing a thick disk and halo distribution toward this direction. The candidate GD-1 stream stars at about  $v_{\rm gsr} \sim -82$  km s<sup>-1</sup> cannot be explained by either a halo or thick disk distribution.

We estimate from Figure 4.3 that about 30 stars in the spectroscopic dataset are from the GD-1 stream. To see the metallicity distribution of the stars in the GD-1 stream, we subtract a scaled version of the histogram with the light line from the histogram with the heavy line. The scaling factor is (115-30)/(1311-30). Since the stars in the velocity selected region contain a smaller fraction of thick disk stars, the subtracted histogram is oversubtracted at high metallcities, and likely undersubtracted at spheroid metallcities. The mean of the stars in the shaded region is [Fe/H]=-1.9, but the real metallicity of the stream is probably somewhat lower than this. Bins with negative counts do not appear in Figure 4.4.

We now return to the sample of stars in Figure 4.2, and select only those of very low metallicity (-2.3 < [Fe/H] < -1.65) in order to isolate stream members from the thick disk and halo field star populations. The low metallicity spectra



Figure 4.4 Histogram of SSPP metallicities for all stars (light line) and for stars with  $-97 < v_{gsr} < -61$  km s<sup>-1</sup> (heavy line). The velocity-selected peak has an excess of stars with metallicity lower than that of the halo. The hashed line indicates a correction to the heavy line for interlopers at other velocities which have the same metallicities as candidate stream stars. This figure suggests that the peak metallicity of the GD-1 stars is lower than [Fe/H]=-1.9.

with positions, colors, and magnitudes that make them candidates for GD-1 stream members are shown in Figure 4.5. Several velocity peaks are now clearly separated from the disk and spheroid. We now examine stars in each of the seven regions numbered in Figure 4.5 and determine their observational properties.

In each region with a clear stream detection, the velocity and velocity dispersion for the GD-1 stream were computed using an iterative method that used only stars within one standard deviation of the mean velocity. We computed the mean and standard deviation of the stars near the velocity peak. Then, we selected stars that were within one standard deviation of the mean and re-computed the mean and standard deviation. The standard deviation calculated this way is an underestimate, since we have removed the tails of the distribution. We corrected the standard deviation assuming Gaussian tails. This process was repeated until the



Figure 4.5 The subset of stars from Figure 4.2 with SSPP metallicities -2.3 < [Fe/H] < -1.65 are presented. Now the counter-rotating GD-1 stream stars stand out more clearly against their field contaminants compared with Figure 4.2. Regions 1,4,5 and 6 have clear peaks; the mean and the error on the mean for these peaks are indicated by the position and height of the rectangles at these four longitude locations. The circles in regions 2, 3, and 7 indicate the area through which the stream should pass if our model is correct. All areas except region 7 seem to have an excess of stars with the expected stream velocities.

computed mean and standard deviation matched the mean and standard deviation used to select the stars in the stream.

#### 4.1.3 The Observed Stream Properties

The observed properties of the stars in the seven GD-1 stream candidate regions are summarized in Table 4.1, where we list region number (N); ; Galactic coordinates (l, b) with errors (we use  $\delta$  to denote a measured error, to distinguish it from the intrinsic dispersion, which we denote with the symbol  $\sigma$ ); the average Galactocentric standard of rest velocity with an error, and the velocity dispersion. The velocity mean and dispersion were calculated as described previously. The tabulated intrinsic dispersions are upper limits to the actual velocity dispersion of the stream; since they are similar in size to the velocity errors for each individual spectrum, the measurement is consistent with an intrinsic velocity distribution of zero. The error in the mean is the velocity dispersion divided by the square root of the number of stars used to compute it. Regions 2, 3, and 7 do not have clear, narrow peaks in the velocity distributions and therefore were not used to fit the orbit, though Figure 4.5 shows there are excess stars at about the right velocities.

Ν	l (°)	b(°)	$\delta b~(^\circ)$	$v_{gsr}$	$\delta v_{gsr}$	$\sigma_v$	$\mathrm{d}_{\mathrm{Sun}}$	$\delta d_{Sun}$	$\mu_l$	$\mu_b$
1	224.47	20.88	0.5	108	5	11	10.4	1.2	7.0	-6.4
2	215.93	30.83	0.2	69			6.5	0.6	8.6	-7.4
3	206.03	40.89	0.2				7.0	0.4	10.0	-7.3
4	197.00	47.54	0.2	-7	1	3.9	7.5	0.3	10.9	-6.5
5	172.30	57.24	0.2	-71	2	5.3	8.0	0.5	11.8	-2.4
6	161.95	59.02	0.2	-87	2	9.2	8.8	0.8	11.5	-0.6
7	99.95	55.00	1.0				9.9	1.2	4.1	5.2

Table 4.1 GD-1 Stream detections, with velocities in km s<sup>-1</sup>, distances in kpc and proper motions in mas  $yr^{-1}$ .

We now calculate our best metallicity estimate for GD-1 by selecting only the 48 stars with spectra in Figure 4.6. We note that these stars were pre-cut on metallicity at an earlier stage (Figure 4.5) to have -2.3 < [Fe/H] < -1.65. A histogram with bins similar to the measurement error yields a GD-1 stream metallicity of  $[Fe/H] = -2.1 \pm 0.1$  dex with a dispersion of  $\sigma = 0.3$  dex (essentially the measurement error). In addition to these statistical errors, there may be systematic errors in the metallicity determinations from SDSS DR7 of ~ 0.2 dex (Allende Prieto et al. 2008).

#### 4.1.4 Orbit Fitting

We now use the data listed in Table 4.1 and the method of Chapter 2 to fit an orbit to the GD-1 stream, assuming the fixed Galactic potential with a logarithmic halo used in Chapter 3. We adopt values of  $v_{halo} = 115$  km s<sup>-1</sup>, q = 1.0 and d = 12 kpc.

Grillmair and Dionatos (2006) postulated the progenitor of this stream is a globular cluster because it has a narrow width in the sky. As a cluster orbits the



Figure 4.6 All stars in regions 1-7 with SEGUE spectra which meet the metallicity, color, magnitude, velocity and proper motion cuts as described in the text are plotted with colored points as indicated in the legend, along with estimated distance to each set of points. Each set of points was shifted to the reference distance of 9 kpc, and overlaid with a M92 fiducial locus, shifted to the same distance moduli. Note the two (blue) points at  $g_0 \sim 15.3$ , which are actually at  $g_0 \sim 14.9$  before shifting. These are candidate BHB stars in the GD-1 stream, at a distance of 7.5 kpc from the Sun in region 4.

Galaxy, stars farther from the progenitor will depart from the orbit due to dynamical friction and scattering of the stream stars. Because the progenitor is presumed to be a compact object with a few km s<sup>-1</sup> velocity dispersion, it is reasonable to assume that the stars in the tidal stream lie approximately on the orbit of the globular cluster (Odenkirchen et al. 2003). Dwarf galaxies, on the other hand, will experience larger spatial dispersions because they have larger dispersions in their energies. Therefore, we fit the orbit to the positions and velocities of the stars in the tidal stream.

To find reasonable initial values for these parameters, we imagine placing a test particle in region 5 at  $(l, b, R_5) = (172.3^\circ, 57.24^\circ, 8.0 \text{ kpc})$ . We then construct a vector between the  $(l, b, R_5) = (172.3^\circ, 57.24^\circ, 8.0 \text{ kpc})$  and  $(l, b, R_6) =$  (161.95°, 59.12°, 8.8 kpc) points. This gives us the direction of the total velocity, because we are assuming the orbit passes through both of these points. The principle initial values for the parameters in region 5 are  $(R_5, v_{x,5}, v_{y,5}, v_{z,5}) = (8.0 \text{ kpc}, -94 \text{ km/s}, -285 \text{ km/s}, -104 \text{ km/s})$ . In practice we start searching for the best parameters in a range of values near these approximate values for the orbital parameters.

#### 4.1.5 Results and Discussion

The metric we wish to minimize is a single stream fitness in  $(\theta = l, \phi = b, v_{gsr}, d_{Sun})$ , given in Equation 2.5. We select five initial sets of parameters and perform the gradient descent to reach the best fit parameters. We then estimate the parameter errors using the Hessian method of Equation 2.18. The best-fit parameters and their errors are  $(R_5, v_{x,5}, v_{y,5}, v_{z,5}) = (8.4 \pm 0.8 \text{ kpc}, -89 \pm 2 \text{ km s}^{-1}, -236 \pm 6 \text{ km s}^{-1}, -115 \pm 3 \text{ km s}^{-1})$ . The chi-squared of this fit is 2.07. The negative velocities indicate a retrograde orbit. The perigalacticon for this orbit is located  $r = 14.43 \pm 0.5$  kpc from the Galactic center at  $(l, b) = (158^\circ, 60^\circ)$ , near region 6. The space velocity of stars in the model at this position is 276 km s<sup>-1</sup>. The apogalacticon is at  $r = 28.7 \pm 2$  kpc from the Galactic center, toward  $(l, b) = (306^\circ, -35^\circ)$ , though we do not observe this direction on the sky. All errors are  $1\sigma$ .

As was demonstrated in Chapter 3, the orbital parameters are fairly insensitive to the choices of parameters in the Galactic potential.

The potential assumed in fitting the orbit is a standard logarithmic flatrotation curve dark matter halo plus a stellar disk. Since the GD-1 stream approaches within 15 kpc of the Galactic center, the effects of the massive disk are felt by the orbit, and increasing the relative mass of the disk vs. the halo can mimic the effects of a flattened halo. At perigalacticon, the disk exerts twice as much gravitational force as the halo. More models and constraints, from this and other streams, are clearly needed to constrain the shape of the dark matter halo.

Figure 4.7 shows the orbit in (l, b) with the stream locations shown. The model prediction is in very good agreement with the experimental observations. The middle and lower panels of Figure 4.7 show the orbit in  $v_{\rm gsr}$  versus l and distance from the Sun versus l. We also see good agreement with the experimental observation.



Figure 4.7 Upper panel: Galactic coordinates. The best fit orbit (with fixed q, d) to the data in the four regions 1,4,5 and 6. Region 1 is leftmost, with  $l = 224.47^{\circ}$ . Galactic (l, b) for all seven regions described in the text are plotted as crosses. Middle panel, plotting Galactocentric radial velocity  $v_{gsr}$  vs. Galactic longitude l for the best fit model and data. Regions 1,4,5 and 6 (left-to-right) have the smallest error bars. Lower panel, plotting Sun-centered distance vs. Galactic longitude l to the stream. The errors on regions 4, 5 and 6 are small; the error bars on the other region data points are limited by how well the position of the stream turnoff (minus a background field) can be identified in a color-magnitude Hess density diagram of stream stars.

Figure 4.8 shows the orbit projected into the three planes of Galactic coordinates (X, Y, Z). We deduce an orbital eccentricity  $e = 0.33 \pm 0.02$  (one sigma error) and an inclination to the Galactic plane of  $i \sim 35 \pm 5^{\circ}$ . Arrows show the relative direction of the stream's retrograde motion compared to the Milky Way.

The final columns of Table 4.1 show the predicted proper motions  $(\mu_l, \mu_b)$  for stars in the stream at each region 1-7 based on the distances in Table 4.1. These predictions may be compared with actual observed proper motions for stream star candidates in Appendix A at each region. In general the agreement is quite good for regions 2-6, given proper motion errors of  $1\sigma = 3$  mas yr<sup>-1</sup> in each coordinate.



Figure 4.8 Shown in heavy black line is the best fit orbit to the four regions 1,4,5 and 6 in Galactic rectangular coordinates (X, Y, Z) in each of the cardinal projections. The coordinate system is a right-handed system with the Sun at at (-8,0,0) kpc and the Galactic center at the origin. The coordinate and turnoff magnitude data from the seven regions is converted to (X, Y, Z) assuming an absolute turnoff F star magnitude of  $M_g = +4.2$ . The seven points are indicated with error bars from distance error estimates. Regions 1 and 7 are indicated in each panel, with the other regions falling in order at intermediate positions. The units of each axis is kpc. An arrow originating at the Sun indicates the direction of Galactic rotation. The arrows associated with the stream indicate the retrograde direction of motion of GD-1 stream stars. The space velocity of the stream at perigalacticon is approximately 276 km s<sup>-1</sup>.

Spectral candidates more than  $2\sigma$  away were excluded from Figure 4.6, dropping about 20% of the candidates, leaving a generally good fit to a shifted M92 fiducial sequence for these regions. Region 7 had fewer good proper motion matches, and it is possible that we are not seeing GD-1 stream candidates here.

Figure 4.9 shows the photometrically selected stars with proper motions available near regions 1,4,5 and 6, along with an equivalent set of field stars (chosen 5 degrees away) for comparison. There's a clear excess of 'on-stream' stars in the



Figure 4.9 For the four regions N=1,4,5,6 where we have fitted the orbit, we select stars within 0.3° of a cubic similar to equation 1 which have measured USNO-B proper motions from the DR7 database. For reference, we select similar sets of field objects offset by 5 degrees in declination from the on-stream objects. We then sub-select stars with colors and magnitudes of stream turnoff candidates and plot the  $\mu_l$  vs  $\mu_b$  of the on-stream (black dots) and off-stream (open circles) for each selected region. There is a clear excess of on-stream points extending to the lower right in each region. We superimpose crosses representing the point where the best fit model (Table 4.1) crosses the stream, and show the derived tangential velocity in the Figure. The cross always falls in the same quadrant with the excess of proper motion points. Typical errors on each point are 3 mas yr<sup>-1</sup> in each direction. The upper and right axes in each figure convert the observed proper motions to tangential velocities, assuming the distance to the stars is the distance to the fitted orbit for that particular GD-1 stream region.

lower right quadrant of each region on-stream – these are likely stream members. The estimated tangential velocities (relative to the Sun) are given.

To search for a possible progenitor, we selected all Milky Way globular clusters from Harris (1996) that had metallicities in the range -2.5 < [Fe/H] < -1.5. Only seven of these globular clusters (Terzan 8, Arp2, NGC 6809, NGC 6749, NGC 6341, NGC 6681, and NGC 6752) are within 5° of the GD-1 orbit. Additionally, we considered NGC 2298, which is  $5.5^{\circ}$  from the orbit, and has a metallicity of [Fe/H] = -1.85. We compared the positions and velocities of these globular clusters with an orbital path that extends all the way around the Milky Way. To create a stream of this length would require a globular cluster to orbit the Milky Way for on the order of gigayears, with the length depending on the concentration of the progenitor, as well as the shape and location of the progenitor's orbit. Of the eight globular clusters, NGC 6809, NGC 6749, and NGC 6752 are ruled out because their distances are more than a factor of two different from the distance to the orbit. The remaining five clusters had radial velocities that are inconsistent with the predicted orbit by more than 50 km/s. We therefore conclude that the Milky Way globular cluster catalog published by Harris (1996) does not contain the progenitor of this stream.

#### 4.1.6 Conclusions

We use spectroscopic kinematic and abundance information to isolate stars in the GD-1 stream, and use the positions and velocities of those stars to derive orbital parameters for its orbit. The GD-1 stream is moving very rapidly on a retrograde orbit around the Milky Way. In the region of the orbit which is detected, it has a distance of about 7-11 kpc from the Sun. The stream's orbit takes it to apogalactic distances of  $28.75\pm2$  kpc, and it has a perigalacticon of  $14.43\pm0.5$  kpc, implying an eccentricity of  $0.33\pm0.02$ . The inclination to the Galactic plane is about  $i \sim 35^{\circ}\pm5$ . The metallicity of the stream is  $[Fe/H] \sim -2.1\pm0.1$  plus systematic errors of a few tenths dex. None of the known globular clusters in the Milky Way have positions, radial velocities, and metallicities that are consistent with being the progenitor of the GD-1 stream.

The consistency between the proper motions of these stream candidates and our best fit model gives us further confidence that we have identified stream members and that our model accurately represents the path on the sky of the stream stars. While we claim only consistency here between the proper motion data and our model, we note that more detailed fits to the proper motion (in addition to the radial velocities) for such nearby streams can be a crucial tool in constraining the halo potential shape and other parameters.

# 4.2 The Cetus Polar Stream

The Cetus Polar Stream was discovered by Newberg, Yanny and Willett (2009) (NYW). Yanny et al. (2009b) noticed a co-moving population of low-metallicity blue horizontal branch (BHB) stars with positions and velocities near, but not coincident with, the Sagittarius trailing tidal tail. The piece of this stream that nearly intersects with the Sgr tidal stream is at  $(l, b) = (140^{\circ}, -70^{\circ})$  and at a distance of 34 kpc from the Sun with a line-of-sight Galactic standard of rest radial velocity  $v_{\rm gsr} = -50 \text{ km s}^{-1}$  and metallicity [Fe/H]  $\approx -2.0$ . In this section, we explore the extent and kinematics of this new stream.

Figure 4.10 shows a color-magnitude diagram of all stellar objects in the South Galactic Cap with zero proper motion and surface gravities of giant stars. Circled observations have the velocity and metallicity we expect for the new tidal stream. The three boxes labeled blue horizontal branch (BHB;  $-0.3 < (g - r)_0 < 0.2, 0.8 < (u - g)_0 < 1.6, 17.7 < g_0 < 18.4$ ), red giant branch (RGB;  $-12.75(g - r)_0 + 25.62 < g_0 < -12.75(g - r)_0 + 27.12, 16.8 < g_0 < 17.8$ ), and lower red giant branch (LRGB;  $0.47 < (g - r)_0 < 0.53, 18.5 < g_0 < 19.7$ ) in Figure 4.10 have a relatively high fraction of stars likely to be in the tidal stream, and comparison with M92 and M3 fiducials from An et al. (2008), shifted to 34 kpc, shows that they are also likely to be from the same stellar population. From the BHB fiducials we extracted from the An et al. (2008) data and distance moduli from Harris (1996), we estimate the absolute magnitude of the BHBs in the color range  $-0.3 < (g - r)_0 < -0.2$ , where most of the BHBs lie, is  $M_{g_0} = 0.45$ .

The upper panel of Figure 4.11 shows the velocities of stars in the three colormagnitude boxes in Figure 4.10. The ones with lower metallicity are circled. The solid outline identifies stars with velocities of the Sgr trailing tidal tail (compare with Law, Johnston & Majewski 2005; Yanny et al. 2009b;  $60^{\circ} < \Lambda_{\odot} < 140^{\circ}$ ). The dashed outline shows velocities of stars in the new stream. At higher Galactic latitude we relied primarily on the locus of low metallicity RGBs to select the velocities of stars in the new stream. The new stream has lower metallicity than those of the Sagittarius trailing tidal tail, as demonstrated by the fraction of larger to smaller, point-like symbols within the upper outlined region compared with the lower region with Sgr velocities.

The lower panel of Figure 4.11 explores the distance to the tidal stream, by showing  $g_0$  vs. b for the stars in the upper plot that are likely stream members, and photometrically selected BHBs in the region of the newly identified Cetus Polar Stream.

We find an approximately linear relationship between  $g_0$  and Galactic latitude  $(g_0 = -0.0162b + 17.09)$  in this portion of the stream. Distances were estimated and assuming  $M_{g_0} = 0.45$ .

Distance estimates are tabulated in Table 4.2, with only statistical errors included. Distance errors may be systematically too high or too low by 10%, depending on the determination of the absolute magnitude of BHBs (Sirko et al. 2004).

The four panels of Figure 4.12 show (upper left) an estimate of the positions of the F turnoff stars in the CPS, and the positions of the photometrically selected BHB stars; (upper right) the (l, b) distribution of spectra with colors and magnitudes similar to those in the CPS; (lower left) the distribution of F turnoff stars in Sgr and the CPS, with the stars with CPS velocities superimposed; and (lower right) the same F turnoff stars with the stars with Sgr stream velocities superimposed. Note that there is an overdensity of photometrically selected BHB stars that lines up with the background-subtracted F turnoff star overdensity, and the CPS velocity-selected BHB, RGB, and LRGB stars, running approximately along Galactic latitude  $l \sim$ 143°. Stars that are velocity selected to be candidate Sgr stream stars follow a different path in the sky, along the Sgr dwarf tidal tail as tabulated in Newberg et al. (2003).

Table 4.2 summarizes the properties of the CPS at four Galactic latitudes, shown in Figures 4.11 and 4.12. In addition to the position, velocity, and distance of the stream as estimated from Figures 4.11 and 4.12, we list the approximate velocity dispersion of the line-of-sight velocities ( $\sigma_v$ ), and the number of spectra at each location. The velocity dispersion for each stripe 76, 79, 82, and 86 is computed from the spectra with  $120^\circ < l < 165^\circ$  that are shown in the lower left panel of Figure 4.12, and tabulated in Table 4.2. Since the intrinsic SDSS/SEGUE radial velocity errors are about 4 km s<sup>-1</sup>, the intrinsic velocity dispersion of the CPS is about 4.5 km s<sup>-1</sup> in stripes 76, 82, and 86, and about 10 km s<sup>-1</sup> in stripe 79.

We fit an orbit to the four CPS locations in Table 4.2, following the procedure outlined in Chapter 2. Due to the polar nature of this stream, the orbit was fit in  $(\theta, \phi) = (b, l)$  space. The best-fit orbit is shown by the solid black lines in Figures 4.11 and 4.12. The orbits shown are for a logarithmic halo with  $v_{halo} = 115$  km s<sup>-1</sup>, q = 1.0 and d = 12 kpc. The best fit kinematics for CPS at  $(l, b) = (144^{\circ}, -71^{\circ})$ are  $(R, v_x, v_y v_z) = (31.1 \text{ kpc}, -103 \text{ km s}^{-1}, 80 \text{ km s}^{-1}, 76 \text{ km s}^{-1})$ . The fitness of this solution is  $\chi^2 = 1.08$ . As expected from GD-1 and the single test streams of Chapter 3, CPS places no useful constraints on the background halo parameters.

l (°)	$\delta l~(^{\circ})$	b(°)	$v_{gsr}(\mathrm{km}\;\mathrm{s}^{-1})$	$\delta v_{gsr} (\mathrm{km \ s^{-1}})$	$d_{\rm Sun}~({\rm kpc})$	$\delta d_{Sun} (kpc)$
144	2	-71	-29.8	6.4	36.1	1.9
144	3	-62	-42.7	6.5	33.8	1.8
142	3	-54	-39.2	11.1	31.8	1.7
142	4	-46	-59.2	5.8	30.1	1.6

 Table 4.2 Cetus Polar Stream detections

The Cetus Polar Stream is, in principle, an extremely useful stream to model. Its polar nature may allow it to be used to constrain the Z-direction flattening. However, this stream has fewer detections than others available for study. Also, the detections have imprecise sky coordinates and are thus less useful for constraining the parameters of the Galactic potential. It is for these reasons that CPS will not be modeled in the simultaneous orbit fit, presented later in this Thesis. In the future, with more precise detections, this stream may provide useful constraints.

### 4.3 The Orphan Stream

The Orphan Stream, co-discovered by Grillmair (2006) and Belokurov et al. (2006), is a stellar stream in the North Galactic Cap that is perpendicular to the leading arm of the Sgr tidal tail. Grillmair (2006) was the first to publish a full discovery paper, showing that the stream was at least  $60^{\circ}$  long and  $2^{\circ}$  wide, and likely the remains of a small dwarf galaxy that has been completely disrupted.



Figure 4.10 We show as small black dots the color and  $g_0$  magnitude for all the SDSS DR7 stellar spectra in the South Galactic Cap  $(b < 0^{\circ})$  with surface gravities of giant stars  $(1 < \log g < 4.0)$ , and essentially zero proper motion  $(|\mu_l| < 6 \text{ mas yr}^{-1}, |\mu_b| < 6 \text{ mas yr}^{-1})$ . These cuts select objects likely to be in the stellar halo. The circles show those points that have velocities and metallicities consistent with membership in the new stellar stream  $(-77 < v_{gsr} < 0 \text{ km s}^{-1}, -4 < [Fe/H] < -1.9$ . Stars with  $-0.3 < (g - r)_0 < 0.2$  are likely BHB stars, so for these stars we used the SDSS  $[Fe/H]_{WBG}$  metallicity measurement (blue circles). The stars with  $0.3 < (g - r)_0 < 0.8$  are likely giant stars, so in this color range we used the SDSS  $[Fe/H]_a$  metallicity measurement (red circles). Fiducial sequences for M92 and M3, shifted to 34 kpc, are shown for reference. Figure and caption from Newberg, Yanny, and Willett (2009).

In Grillmair's paper, the stream was characterized as about 21 kpc distant from the Sun along the whole length of the stream. Belokurov et al. (2007) published an independent discovery paper naming the Orphan Stream; they found a similar length, width, and likely origin. However, Belokurov et al. found a strong distance gradient, from 20 kpc at one end of the stream to 32 kpc from the Sun at the other end. They also published "suggestive" velocities from sparse samples of Sloan



Figure 4.11 Top panel shows Cetus Polar Stream candidate radial velocities as a function of Galactic latitude b. Spectra with metallicities of -4 < [Fe/H] < -1.9 are circled. The dashed boxed region shows the velocities of CPS stars, while the solid boxed region shows those of the Sgr dwarf trailing tidal tail. We overlay calculated velocities of thick disk stars with  $50^{\circ} < l < 190^{\circ}$  (top to bottom) in magenta to show there is no confusion with stream candidates. Filled circles in the lower panel show the apparent magnitudes of the stars in the upper panel that have metallicities and velocities expected of the CPS. The triangles show photometrically selected BHB stars (see Yanny et al. 2000 for selection technique) for  $120^{\circ} < l < 165^{\circ}$ . The trend with b is consistent between photometrically and spectroscopically selected BHBs. The solid black line in both panels shows the best fit orbit to the CPS. Figure and caption from Newberg, Yanny, and Willett (2009).



Figure 4.12 The upper left panel shows the density of turnoff stars in a CPS color magnitude box  $(20.25 < g_0 < 21.5, 0.22 < (g - r)_0 < 0.36)$  minus the density of stars in a Sgr box  $(20.35 < g_0 < 21.85, 0.10 < (g - r)_0 < 0.20)$  in polar Galactic coordinates, origin at the SGP. The overdensities (dark areas) running along  $l = 140^{\circ}$ from  $b = -70^{\circ}$  up to  $b = -40^{\circ}$  show the CPS. The blue dots in the upper left panel show the positions (offset  $3^{\circ}$  in b for clarity) of photometrically selected CPS candidate BHBs, with  $-0.0162b + 16.94 < g_0 < -0.0162b + 17.24$ . Note the excess along  $130^{\circ} < l < 150^{\circ}, -70^{\circ} < b < -40^{\circ}$ . The upper right panel shows (magenta circles) the locations of stars with SDSS/SEGUE spectra in the color-magnitude selection boxes of Figure 4.10, showing the completeness coverage of the spectroscopy relative to the imaging. The lower left panel shows the density of turnoff stars with  $20.5 < g_0 < 22.5, 0.26 < (g - r)_0 < 0.30, (u - g)_0 > 0.4$ , highlighting both the Sgr and Cetus debris streams (the Sgr tidal stream is more prominent). The filled circles show the stars with velocities and metallicities consistent with membership in the new CPS, color coded by spectral type. The heavy black curve shows the best fit orbit for the CPS structure. The lower right panel shows (crosses) the positions of low metallicity stars with spectra in the upper panel of Fig. 4.11 that have the velocities of the Sgr trailing tidal tail (the low metallicity subset of SDSS/SEGUE Sgr spectra), along with a Sgr locus (heavy line). Figure and caption from Newberg, Yanny, and Willett (2009).

Digital Sky Survey (SDSS) data in two fields, ranging from  $V_R = -40$  km s<sup>-1</sup> at the close end of the stream to ~ 100 km s<sup>-1</sup> at the distant end of the stream. They noted that the dwarf galaxy Ursa Major II, the HI clouds of Complex A, and a number of anomalous globular clusters (including Ruprect 106 and Palomar I) lie near the same great circle as the Orphan Stream stars.

Newberg, Willett, Yanny and Xu (NWYX, 2010) performed an externely detailed re-extraction of the Orphan stream in BHBs. The stream, as it appears on the sky, is shown in F-turnoff stars in Figure 4.13. Their work is relevant to this Thesis in two regards: the orbit fits to the Orphan Stream, as well as the Orphan Stream stellar density in F-turnoff stars, shown in Figure 4.14. We will not fully reiterate this work here, only those parts directly applicable to this Thesis will be presented.

NWYX obtained Orphan Stream detections which are given in Tables 4.3 and 4.4. These detections were fit using the methods outlined by the previous chapter in a variety of Galactic gravitational backgrounds. Two main questions needed to be answered: can the Orphan Stream constrain the Galactic halo mass and does the disk model affect the fits? To answer these we performed orbit fits in seven Galactic models:

- 1. We fit the exact model of Xue et al. (2008), using an exponential disk and NFW halo. The parameters for bulge, disk, and halo were taken from their paper. the mass of the bulge, disk and halo, integrated out to 60 kpc, are tabulated in Table 4.5. The mass of the Milky Way using this model is  $M(R < 60 \text{ kpc}) = 4.0 \times 10^{11} \text{M}_{\odot}$ , of which  $3.3 \times 10^{11} M_{\odot}$  is in the halo. The value for the scale radius,  $r_s = 22.25$  kpc, used for all the NFW potentials in this work comes from the top panel of Figure 16 of Xue et al. (2008). For a Milky Way virial radius of  $r_{\text{vir}} = 240$  kpc (Sales et al. 2007), this corresponds to a concentration index of  $c \sim 11$ .
- 2. We fit the same model as the previous case, but allow the  $v_{c,max}$  normalization of the NFW model (the halo mass) to vary in amplitude.
- 3. We fit the same bulge and exponential disk model as the previous two cases,


Figure 4.13 Stars with  $20.7 < g_{\rm corr} < 21.7$ , the magnitude centered on the Orphan Stream's turnoff, are plotted in a Hess diagram over the SDSS footprint. The stream can be seen extending to nearly  $l = 170^{\circ}$ . Additionally,  $B_J$ ,  $R_2$  data from SuperCOSMOS is extracted and added to the figure to cover the region of the Orphan Stream where no SDSS data exists. The stars here have  $20 < B_J < 21, 0.5 < B_J - R_2 < 1.0$  (with the default calibration and reddening corrections adopted throughout). A narrow trail is visible in the SuperCOSMOS extension, running from  $(l, b) = (268^{\circ}, 38^{\circ})$  to  $(255^{\circ}, 48^{\circ})$ . The lower panel adds the  $B_{\rm corr} = 0$  trace of the stream, which differs slightly from  $B_{\rm Orphan} = 0^{\circ}$  for  $l < 200^{\circ}$ . The location of the halo object Segue-1 is indicated. The inset shows a blowup of the region with  $230^{\circ} < l < 280^{\circ}, 30^{\circ} < b < 70^{\circ}$ .



Figure 4.14 Number counts of F turnoff stars within  $\pm 2^{\circ}$  of the stream are plotted as an open black histogram. The number of background turnoff stars off-the-stream on either side ( $2^{\circ} < |B_{\rm corr}| < 4^{\circ}$ ) are plotted in red. The difference is plotted as a hashed histogram. Note the significant excess of turnoff stars over background near  $\Lambda_{\rm Orphan} = +23^{\circ}$ , corresponding to (l, b) = ( $255^{\circ}, 49^{\circ}$ ). It is possible that the stream progenitor lies in this region of sky. The bins at  $\Lambda_{\rm Orphan} = 21^{\circ}, 33^{\circ}$ , and  $36^{\circ}$  have been corrected for incompleteness in both the data and background counts.

but use a logarithmic halo model. We allow the normalization (mass) of the halo to vary as a free parameter to be fit.

- 4. We fit stream kinematics only, assuming a spherical logarithmic halo as given by Law et al. (2005). Disk and bulge parameters for this case are identical to the Law et al. (2005) paper. Their model, with a disk model from Miyamoto and Nagai (1975), yields  $M(R < 60 \text{ kpc}) = 4.7 \times 10^{11} M_{\odot}$ .
- 5. We fit the same model as case 4, but allow the halo speed above to vary as a free parameter within the spherical logarithmic halo potential model, while keeping d = 12 kpc, where d is the halo core softening radius.

- 6. We fit the kinematics within the spherical NFW halo given in Navarro et al. (1996) and further described by Klypin et al. (1999), normalized to  $M(R < 60 \text{ kpc}) = 3.3 \times 10^{11} \text{M}_{\odot}$ . We fit the  $v_{c,max}$  of the NFW model to give a similar total potential to that for the logarithmic halo with  $v_{\text{halo}} = 114 \text{ km s}^{-1}$  above. These give rise to  $v_{c,max} = 155 \text{ km s}^{-1}$  and  $r_s = 22.25 \text{ kpc}$ .
- 7. We fit the same model as case 6, but allow the NFW maximum circular speed  $v_{c,max}$  to vary while keeping  $r_s = 22.25$  kpc.

l	b	$\delta b$	$\Lambda_{Orphan}$	$B_{Orphan}$	BHB $g_0$	$\delta g_0$
255	48.5	0.7	22.34	0.08	17.1	0.1
245	52.0	0.7	15.08	0.56		
235	53.5	0.7	8.86	0.21		
225	54.0	0.7	2.95	-0.23	17.6	0.2
215	54.0	0.7	-2.93	-0.33	17.9	0.1
205	53.5	0.7	-8.85	-0.09	18.0	0.1
195	52.0	0.7	-15.08	0.05		
185	50.5	0.7	-21.42	1.12	18.6	0.1
175	47.5	0.7	-28.59	1.88		
171	45.8	1.0	-31.81	2.10		

Table 4.3 Orphan Stream photometric detections

$\Lambda_{Orphan}$	l	b	$\delta b$	BHB $g_0$	$\delta g_0$	$v_{\rm gsr}$	$\delta v_{\rm gsr}$	$\sigma_v$	Ν	d	$\delta d$
-30	173	46.5	0.7	18.8	0.2	115.5	6.7	11.5	4	46.8	4.5
-20	187	50.0	1.0	18.5	0.1	119.7	6.9	11.9	4	40.7	1.9
-9	205	52.5	0.7	18.0	0.1	139.8	4.6	12.9	9	32.4	1.5
-1	218	53.5	1.0	17.8	0.1	131.5	3.1	8.2	8	29.5	1.4
8	234	53.5	0.7	17.4	0.1	111.3	11.1	11.1	2	24.5	1.2
18.4	249.5	50.0	0.7	17.1	0.1	101.4	2.9	9.8	12	21.4	1.0
36	271	38.0	3.5	16.8	0.1	38.4	1.7	2.5	3	18.6	0.9

Table 4.4 Orphan Stream spectroscopic detections

To obtain our initial guess for the kinematic parameters, we draw from Tables 4.3 and 4.4 a test particle at Point 1:  $(l, b, R) = (218^{\circ}, 53.5^{\circ}, 30 \text{ kpc})$  and construct a vector to Point 2:  $(l, b, R) = (215^{\circ}, 54.0^{\circ}, 31 \text{ kpc})$ . By matching the radial velocity at Point 1, we obtain an initial parameter starting point of  $(R, v_x, v_y, v_z) = (30 \text{ kpc}, -125 \text{ km s}^{-1}, 75 \text{ km s}^{-1}, 95 \text{ km s}^{-1})$ .

With this initial kinematic guess we will consider seven specific cases, which include three potential models published by previous authors, and four models in which the halo mass is varied to best match our data (logarithmic and NFW halos are compared, as well as a low mass exponential disk vs. a high mass M-N disk):

The models were fit to all of the available data in Tables 4.3 and 4.4, with the exception of the last point in Table 4.4. Attempting to fit the data set with the  $l = 271^{\circ}$  point included resulted in substantially worse  $\chi^2$  values ( $\chi^2 \sim 4$ ). Additionally, fitting this point introduced deviations of the orbit from the observed stream distances, with the high l distances being underestimated, and the low ldistances being overestimated. These systematic differences between the model and data led us to perform all of our best fit models without the  $l = 271^{\circ}$  point, for all six of the orbit fits listed above.

In addition to these cases, we also attempted several more which did not produce very interesting results, but which we will summarize here. We ran several orbits fits in an attempt to measure the halo flattening parameter, q. The fits were very insensitive to q; fitting this parameter in the logarithmic halo yielded results with a very large error in q. Therefore, we fixed the flattening parameter at q = 1.

We also attempted to simultaneously fit  $v_{halo}$  and d, and  $v_{c,max}$  and  $r_s$ , for the logarithmic and NFW halos, respectively. In general, fitting the scale lengths did not substantially change the halo speed results, and the scale lengths were fit only with very large errors, on the order of tens of kiloparsecs. Therefore, we did not attempt to fit the scale lengths in either model.

### 4.3.1 Results

For each halo model, we ran five random starts of the gradient search fitting algorithm in the same manner described in Chapter 2. Once the best fit orbit was found for each model, we computed the Hessian errors by varying the parameter step sizes until the errors converged for different step size choices. Using a starting point of  $(l, b) = (218^\circ, 53.5^\circ)$ , the best fit parameters and their errors are enumerated in Table 4.5.

Figure 4.15 shows the three orbit fits for the models with the exponential disk.

The position and velocities of the datapoints are well fit by all three exponential disk models. The lower panel of Figure 4.15 shows that for this stream, the distance fit is most sensitive to the potential. The weaker potential of the lower mass halo model gives the best fit to observed stream distances from Tables 1 and 2 (black points with error bars), as it allows the stream to escape to larger Galactocentric radii without significantly changing the observed radial velocity gradient. There is little difference between the best fit NFW halo and the best fit logarithmic halo.

Figure 4.16 shows the circular velocity curve for models N=1-3 from Table 4.5, which use the same exponential disk parameters as Xue et al. (2008) in all cases. The figure shows that the orbits that are well fit to the Orphan Stream don't fit the Xue et al. (2008) nor the Koposov et al. (2009) result; the rotation speed of the model is too low at all Galactic radii.

We therefore try using a M-N disk with parameters as adopted by Law et al. (2009). This disk is about twice massive than the exponential disk model. We note here that we have not fully explored all possible disk shapes and masses, and a more massive exponential disk may give similar results to the more massive M-N disk considered here. Figure 4.17 shows the four fits for the models with the M-N disk. As before, lower halo speeds are a better fit to the Orphan Stream than those used by previous authors, and there is little difference between the fits for the best fit NFW and logarithmic halos. Evidently, the Orphan Stream is telling us that the mass of the Milky Way is smaller than previously thought, so that distant portions of the stream, which are therefore experiencing lower gravitational attraction to the Galaxy, are pushed further away from the Galaxy center at a given energy.

Figure 4.18 shows the same circular velocity as Figure 4.16 for models 4-7 with an M-N disk. This time we find that the low mass halo orbit fits *are* a good fit to the Xue et al. (2008) and Koposov et al. (2009) result. They actually fit the Xue et al. (2008) model better than the model fit in that paper. Using the higher velocity leads to a systematic deviation in the distances, so that the model is further away than all of the datapoints at high Galactic latitude, and closer for low Galactic latitudes. Also, as we will see later, adding velocity segregation from an N-body simulation only makes the systematic errors in the distance larger; for example stars



Figure 4.15 Orbit fits  $(b, v_{\rm gsr})$ , and d vs. Galactic longitude) to the Orphan Stream data for three different halo potentials (models 1-3), with a fixed bulge and exponential disk model with parameters copied from Xue et al. (2008). In red (dot dash, model 1) we show the best orbit using the NFW halo parameters from Xue et al. (2008). In orange (dotted, model 2) we show the best fit if we allow the  $v_c$  parameter to vary in the NFW profile. In black (solid, model 3) we show the best fit if we instead use a logarithmic potential. Note that the best fit NFW and logarithmic potentials give similar fits, but both have a lower velocity (and therefore halo mass) than found by Xue et al. (2008). To fit the Orphan Stream distance data, it is necessary to reduce the amplitude of the halo potential by about 40%. The distance-velocity space locations of Segue-1, Ursa Major II, Complex A are indicated as labeled in the top panel. Note that only Segue-1 is possibly associated with this tidal stream.



Figure 4.16 We now compare the best fit models from Figure 4.15 with the Milky Way rotation curve data from Xue et al. (2008) and Koposov et al. (2009). The red curve (dot dash, model 1) is the best fit found by Xue et al. (2008), from Figure 16a of that paper, so it is a good fit to the data points. The orange curve (dotted, model 2) and black curve (solid, model 3), for which we allowed the halo mass to vary, are significantly below all of the rotation curve data points. The rotation curves from individual components of the potential (exponential disk, bulge, and NFW halo) used in Xue et al. (2008) are also shown. We also show a Miyamoto-Nagai disk, scaled to a total mass similar to that of the exponential disk, to show that the rotation curve in the region we probe is not dramatically different for different disk profiles, and the halo rotation curves for the lower mass halos.

in the leading tidal tail have lower total energy than the progenitor and have smaller Galactocentric distances. The energy difference increases as a function of distance from the progenitor, along the stream. However, we note that our formal error bar on  $v_{\rm halo} = 73 \pm 24$  km s<sup>-1</sup> is quite large and marginally consistent (within 2 sigma) with the higher  $v_{\rm halo} = 114$  km s<sup>-1</sup> value of Law et al. (2005) and others. Although the formal error bars are large, the orbit fits suggest that a lower halo speed is preferred.



Figure 4.17 Orbit fits  $(b, v_{gsr}, \text{ and } d \text{ vs. Galactic longitude})$  to the Orphan Stream data for potential models 4-7, with a fixed bulge and Miyamoto and Nagai (1975) disk with parameters copied from Law et al. (2005). In green (dash, model 4) we show the fit best orbit using the logarithmic halo parameters from Law et al. (2005). In red (dot dash, model 6) we show the best fit with NFW halo parameters fixed from Xue et al. (2008). Neither of these fixed halo potentials from previous papers gives a good fit to the relative distances along the Orphan Stream; the slope of the d vs. l plot is too shallow for these models. The black (solid, model 5) and orange curves (dotted, model 7) show the best fit orbits if the disk is fixed and the halo masses are allowed to vary for the logarithmic and NFW profile halos, respectively. As in Figure 4.15, we see very little difference between the best logarithmic and NFW profile fits, and the preferred values of the halo mass is lower than that of previous authors. Again, it is necessary to reduce the amplitude of the halo potential by about 40% to fit the Orphan Stream distances. The locations of Segue-1, Ursa Major II, Complex A are as labeled as in Figure 4.15.



Figure 4.18 We now compare the best fit models from Figure 4.17 with the Milky Way rotation curve data from Xue et al. (2008) and Koposov et al. (2009). All of the models use a fixed M-N disk and bulge, with a mass about twice as large as the exponential disk in the models in Figures 4.15 and 4.16. The green (dash, model 4) and red (dot dash, model 6) curves use fixed parameters from the logarithmic halo potential of Law et al. (2005) and NFW halo potential of Xue et al. (2008), respectively. Neither of these fit the Orphan Stream distances, and they are not especially good fits to the rotation curve, predicting rotation velocities above most of the data points. The black (solid, model 5) and orange curves (dotted, model 7) are reasonably good fits to the Xue et al. (2008) data, even though they were not fit to this data. In these models, the halo mass was allowed to vary. Our best fit model is model 5 (black, solid curve), though model 7 is nearly as good. The rotation curves from individual components of the potential (M-N disk, bulge, and NFW or logarithmic halo) are also shown.

Of the models that we tried, the three low-mass exponential disk models (1-3) fit either the Orphan Stream data or the Xue et al. (2008) rotation curve, but not both. The two low halo mass models (models 5,7) with a higher mass M-N disk fit the best, with a logarithmic halo fitting about the same as an NFW profile. The two higher halo mass models (models 4,6) are poorer fits to both the Orphan Stream and the Xue et al. (2008) rotation curves.

Although we cannot formally rule out the halo models fit by Xue et al. (2008) and Law et al. (2005), in order to simultaneously fit the Xue et al. (2008) circular velocity data, we prefer a lower total mass of the Milky Way than measured in either of those two papers. Our total Milky Way mass is 60% of that found by Xue et al. (2008) and Law et al. (2005). As mentioned earlier, our masses are at the low end, but not out of range of recently published masses. To estimate the virial mass of the Milky Way given our best fit model 5, we multiply the halo mass within 60 kpc by 4 (for a virial radius of 240 kpc), and add the disk plus bulge mass. The result is  $M(R < 240 \text{ kpc}) = 6.9 \times 10^{11} M_{\odot}$ . Of interest is the recent result of Odenkirchen et al. (2009), who suggested that a lower halo mass might make it easier to fit the kinematics of the Pal 5 globular cluster stream. The findings on Milky Way circular velocity with radius to distance of  $R \sim 60$  kpc by Xue et al. (2008) also are consistent with our low value for the halo speed. When the M-N disk model is used, they are also in rough agreement with Koposov et al. (2009) who measure  $v_c(R = 8 \text{kpc}) = 224 \pm 13 \text{ km s}^{-1}$ .

We note that we didn't explore scaling the mass of the disk (or bulge) beyond their default values of  $M_{\text{disk}} = 10^{11} M_{\odot}$  for the M-N disk or  $M_{\text{disk}} = 5 \times 10^{10}$  for the exponential disk. Thus, while we find that the heavier M-N disk (combined with a lower amplitude halo) fits the joint data sets of the Orphan Stream, and the Xue et al. (2008) BHBs circular velocity points, better than models with the lighter exponential disk, it is likely that similar fits could be obtained with a heavier exponential disk (though still with a lighter halo). If more data were available, the disk mass could be varied to find the best combination of disk and halo mass.

It is important to note from Figures 4.15 and 4.17 that if we could follow the Orphan Stream just a little farther out into the halo, we would have a much better



Figure 4.19 The black line shows our preferred fit orbit in the logarithmic halo with  $v_{\text{halo}} = 73 \text{ km s}^{-1}$  in right-handed Galactic rectangular coordinates (X, Y, Z). The Sun is at (-8, 0, 0) kpc and the Galactic center at the origin. The (l, b) coordinates and BHB magnitudes are converted to (X, Y, Z) assuming a BHB absolute magnitude of  $M_g = 0.45$ . The arrows indicate the forward direction of the orbit and the Sun's motion. Observations of the Orphan Stream data are shown as asterisks. The positions of Segue-1, UMa II and Complex A are shown with the same symbols as in Figure 4.17.

Figure 4.19 shows the best fit  $v_{halo} = 73 \text{ km s}^{-1}$  logarithmic halo model in (X,Y,Z) Galactic coordinates with the direction of motion indicated by the arrows. From the apo- and peri-galactic distances of ~ 90 kpc and ~ 16.4 kpc, respectively, we calculate an orbital eccentricity of e = 0.7. The inclination of the Orphan Stream is  $i \sim 34^{\circ}$  with respect to the Galactic plane as seen from the Galactic center. It will be important to obtain data for the Orphan stream at apogalacticon in order to distinguish between different models for the Galactic potential.

### 4.3.2 The Orphan Stream is moving in a prograde direction

The velocities of the fit orbit definitively show that the stream stars are moving in a direction from higher l to lower l. As a simple check, if we reverse the sign of all the velocities  $(v_x, v_y, v_z)$  in the model orbit, the path on the sky that the stream traces out is the same, but the radial velocities do not match the observations by a wide amount  $(|\delta(v)| > 100 \text{ km s}^{-1})$ . Thus the Orphan Stream is on a prograde orbit around the Milky Way. One may now ask if the visible piece of the Orphan tidal stream is a leading or trailing tidal tail. If we assume that the 'progenitor' of the Orphan Stream lies in the range  $248^{\circ} < l < 268^{\circ}$  as the density plot of Figure 4.14 suggests , (with the density enhancement visible at  $(l, b) = [253^{\circ}, 49^{\circ}]$ ), then the portion of the tidal stream stretching from  $l = 250^{\circ}$  to  $l = 170^{\circ}$ , combined with the space velocity of stars moving in that same direction, implies that we are seeing a leading tidal arm, rather than a trailing tidal arm. The piece of stream intercepted at  $l = 270^{\circ}$  is then a trailing tidal arm.

## 4.3.3 N-Body Realization

We now comment on the fact that our 'simple orbit integration and fitting' technique is not an N-body simulation, and therefore effects of stars leaving the progenitor and drifting ahead or behind to greater and lower energies, which changes their distance from the central body, are not captured as they would be in a full N-body.

To conduct the N-body simulation, we first integrate the log halo (best fit model 5) orbit back 4 Gyr and place a 10,000 particle Plummer sphere at the predicted location. We use a Plummer sphere scale radius of  $r_s = 0.2$  kpc and total mass of  $M_{\text{total}} \sim 2.5 \times 10^6 M_{\odot}$ . Using the orbit kinematics from 4 Gyr in the past, we evolve it forward for 3.945 Gyr so that the dwarf progenitor ends up at  $l \sim 250^{\circ}$ . We conducted the N-body simulation using the *gyrfalcON* tool of the NEMO Stellar Dynamics Toolbox (Teuben, 1995).

The result of this simulation is given in Figure 4.20. The top three panels show the logarithmic halo orbit with  $v_{\text{halo}} = 73 \text{ km s}^{-1}$  and N-body results for (l, b),  $(l, v_{\text{gsr}})$ , and  $(l, d_{\odot})$ .

We see that the N-body stream is a good fit to the sky locations and velocities of the simple orbit (black curve in Figure 4.20). In the distance comparison (third panel), the N-body is significantly below the orbit at low l. This is in agreement with our intuition for a stream that is moving from high l to lower l, since energy differentials due to the internal dispersion of stars in the progenitor draw stars with lower energy (and lower Galactocentric distance) further ahead closer to the Galactic center than those with higher total energy. Note that this is independent of the question of whether the tail is leading or trailing. We know the direction that the stream is going from the observed radial velocities and distances. We expect that we are seeing a leading tidal tail because we think the dwarf remnant is located at the high Galactic longitude end of the observed portion of the stream. But even if we are incorrect about the position of the dwarf, the observed slopes in the lower panels of figures 12 and 14 indicate a lower mass Milky Way, and a full N-body simulation will only make the slope for higher mass estimates fit worse.

The lowest panel in Figure 4.20 shows the density of N-body simulated particles remaining at the current epoch along the stream vs. l. This density distribution may be compared with that of the data in Figure 4.14. The N-body simulation has a lower density of particles near  $l = 240^{\circ}$ , ( $\Lambda_{\text{Orphan}} = 15^{\circ}$ ). These two effects are apparent in the Orphan imaging data in Figures 4.14 and 4.13, respectively. This suggests that our method of fitting a simple orbit to the tidal debris is a reasonable first approximation, and fitting the density is a useful diagnostic tool.

### 4.3.4 Conclusions

We summarize our findings as follows:

The Orphan Stream data is best fit to a Milky Way potential with a halo plus disk plus bulge mass of about  $2.6 \times 10^{11} M_{\odot}$ , integrated to 60 kpc from the Galactic



Figure 4.20 The *b*,  $v_{\rm gsr}$  and  $d_{\rm Sun}$  vs. l (top 3 panels) orbit of the preferred halo (logarithmic potential with  $v_{\rm halo} = 73$  km s<sup>-1</sup>, M-N disk and heavy bulge), simply integrated, is shown as a heavy black curve. A 10,000 point N-body simulation of a Plummer sphere dwarf with  $M_{\rm total} \sim 10^6 M_{\odot}$  is integrated in this potential from a time 4 Gyr ago forward for 3.945 Gyr placing the proposed progenitor at  $(l, b) \sim$  $(250^\circ, 50^\circ)$ . The lower panel shows the density distribution of the final (current) epoch positions of the N-body points as a histogram in *l*. Note the similarity of the the simulated distribution, including the dip in density at  $l = 240^{\circ}(\Lambda_{\rm Orphan} = +15^{\circ})$ seen in Figure 4.14, and the spread (first panel) and density falloff at  $l < 180^{\circ}$  seen in Figure 4.13.

center. Our best fit is found with a logarithmic halo speed of  $v_{\text{halo}} = 73 \pm 24 \text{ km s}^{-1}$ , a disk+bulge mass of  $M(R < 60 \text{ kpc}) = 1.3 \times 10^{11} M_{\odot}$ , and a halo mass of  $M(R < 60 \text{ kpc}) = 1.4 \times 10^{11} M_{\odot}$ . However, we can find similar fits to the data that use an NFW halo profile. Our halo speed of  $v_{\text{halo}} = 73 \pm 24 \text{ km s}^{-1}$  is smaller than that of previous literature. Although our fits with smaller disk masses and correspondingly larger halo masses are not good fits the the Xue et al. (2008) rotation curves, we have not tried enough models to rule out this possibility. Distinguishing between different classes of models requires data over a larger range of distances.

The Orphan Stream is projected to extend to 90 kpc from the Galactic center, and measurements of these distant parts of the stream would be a powerful probe of the mass of the Milky Way.

An N-body simulation yields an excellent fit to the observed data, including matching the approximate shape of the stellar stream star density along the stream. The N-body mass used was about  $M_{\text{total,Orphan}} = 10^6 M_{\odot}$ , about  $10^{-3}$  the total mass of the Sgr stream and dSph system. The total Orphan system mass is not highly constrained.

The list of possible halo objects associated with the Orphan Stream is reduced to one: The 'dissolved star cluster' Segue-1. Other possible associated objects, namely UMa II, the Complex A HI cloud, and the halo's globular clusters are not close to the Orphan Stream orbit presented here in distance or velocity (or both).

Because the orbit fit is not able to significantly constrain the flattening q of the halo potential, we assume a q = 1.0 throughout. Nevertheless, the analysis of the Orphan Stream shows us that we are able to constrain another important halo potential parameter, namely the amplitude of the halo potential. A spherical logarithmic potential with 60% of the mass of the fit of Xue et al. (2008) provides a reasonable fit to the Orphan Stream data.

Ν	${}^{M_{ m Bulge}}_{10^{10}} {}^{M_{ m Sun}}$	Disk	$M_{ m disk}$ $10^{10} M_{Sun}$	Halo	$M_{ m halo}$ $10^{10} M_{Sun}$	$d/r_s$ kpc	$R_{ m fit}$ kpc	$v_x$ km s <sup>-1</sup>	$v_y$ km s <sup>-1</sup>	${v_z \over { m km s}^{-1}}$	$v_{ m halo}$ km s <sup>-1</sup>	$\chi^2$	${}^{M_{60}}_{10^{10}\ M_{Sun}}$
1	1.5	Exp	5	NFW	33	22.25	28.8	$-170 \pm 11$	$94 \pm 2$	$108 \pm 10$	155	1.55	40
2	1.5	Exp	5	NFW	20	22.25	28.5	$-157 \pm 10$	$78 \pm 12$	$107 \pm 9$	$120 \pm 7$	1.37	24
3	1.5	Exp	5	Log	17.6	12	28.4	$-152 \pm 12$	$72 \pm 12$	$106 \pm 9$	$81 \pm 12$	1.35	26.5
4	3.4	M-N	10	Log	35	12	28.9	$-179 \pm 11$	$106 \pm 2$	$109 \pm 10$	114	1.98	47
5	3.4	M-N	10	Log	14	12	28.6	$-156 \pm 10$	$79 \pm 1$	$107 \pm 9$	$73 \pm 24$	1.70	26.4
6	3.4	M-N	10	NFW	33	22.25	28.9	$-178 \pm 5$	$106 \pm 3$	$108 \pm 10$	155	1.96	43.5
7	3.4	M-N	10	NFW	16	22.25	28.7	$-161 \pm 11$	$85 \pm 1$	$107 \pm 9$	$109 \pm 31$	1.73	28.4

Table 4.5 Orphan Stream models.  $M_{60}$  is the total Galaxy mass enclosed within 60 kpc of the Galactic center.

# 4.4 The Sagittarius Dwarf Tidal Stream (Sgr)

The Sagittarius Dwarf Galaxy was discovered by Ibata et al. (1994) while examining horizontal branch and carbon stars near the Galactic center using the 3.9 meter Anglo-Australian Telescope (AAT). The dwarf is on the opposite side of the Milky Way from the Sun and slightly below the Galactic plane at coordinates (l, b) = $(5.6^{\circ}, -14.2^{\circ})$ . In the original discovery paper, Ibata observed that the dwarf is elongated in the direction of the Galactic center, suggesting tidal disruption. Yanny & Newberg et al. (2000) discovered structure in A-colored stars in the Galactic halo using SDSS commissioning data. Simultaneously, Ibata et al. (2001) announced the discovery of carbon star structure in the same place found by Yanny & Newberg et al. (2000). This structure is tidal debris from the disrupted dwarf, and has been subsequently observed by Yanny & Newberg et al. (2000), Newberg et al. (2003), Newberg et al. (2007), Bellazzini et al. (2003), Majewski, et al. (2003), Majewski, et al. (2004), Martínez-Delgado et al. (2004), Fellhauer et al. (2006), Chou et al. (2007) and modeled by Johnston et al. (1995), Ibata et al. (1997), Gómez-Flechoso et al. (1999), Helmi & White (2001), Law et al. (2004), Law et al. (2005), Cole, et al. (2008), Law & Majewski (2010) and Cole (2009).

Plentiful observations of Sgr give us a good starting point for a simultaneous orbit fit. In this work, we will utilize Sgr stream centers and distances found by Cole (2009), and radial velocities given in Law, et al. (2005). So in a manner similar to the Orphan Stream, we have two sets of data: one that is "photometric" and provides sky coordinates and distance, and the other which provides Galactic standard of rest radial velocities. The stream centers and distances are given in Table 4.6 and the velocities are given in Table 4.7.

Spanning the entire sky, the Sagittarius stream has provided an excellent test bed for modeling the structure of the Galaxy. Since its discovery, models have had a mixed success rate. Prior to the discovery of the tidal stream, Johnston et al. (1995) were not able to place tight constraints on the orbit of the dwarf despite extensive numerical simulation. Once the stream was discovered, Majewski et al. (2004, 2005) mapped it extensively in giant stars from 2MASS, formulated an orbital plane, and placed constraints on the radial velocities of the leading and trailing tails.

$\Lambda_{Sgr,GC}$	$B_{Sgr,GC}$	$\delta B_{Sgr,GC}$	d	$\delta d$	
201.16	11.25	5	16.767	0.54	L
205.00	11.01	5	18.138	0.357	$\mathbf{L}$
211.02	9.89	5	21.218	0.509	$\mathbf{L}$
217.63	8.73	5	22.784	0.463	$\mathbf{L}$
221.69	9.03	5	23.491	0.302	$\mathbf{L}$
227.78	8.54	5	25.298	0.408	$\mathbf{L}$
233.18	8.51	5	25.696	0.394	$\mathbf{L}$
248.28	3.97	5	28.296	0.416	$\mathbf{L}$
255.77	3.06	5	31.162	0.522	$\mathbf{L}$
269.08	1.89	5	36.908	0.334	$\mathbf{L}$
279.42	1.76	5	40.365	6.474	$\mathbf{L}$
293.38	-1.33	5	43.449	0.482	$\mathbf{L}$
87.37	-0.01	5	26.077	0.178	Т
105.11	-1.14	5	29.218	0.197	Т
114.42	-2.95	5	30.225	0.250	Т

Table 4.6 Sagittarius Stream photometric detections from Cole (2009). Leading and Trailing tails are denoted L and T respectively.

$\Lambda_{Sgr,GC}$	$v_{gsr}$	$\delta v_{gsr}$	
0.125	171	5	L
293.4	38	10	L
287.2	16.4	10	$\mathbf{L}$
283.6	-18	10	$\mathbf{L}$
275.9	-39.6	10	$\mathbf{L}$
268.2	-58	10	$\mathbf{L}$
264.1	-69.2	10	$\mathbf{L}$
254.3	-84.4	10	$\mathbf{L}$
248.7	-97.2	10	$\mathbf{L}$
229.1	-109.2	10	$\mathbf{L}$
131.9	-170	10	Т
114.9	-135.6	10	Т
99.0	-106	10	Т
87.7	-85.2	10	Т
75.9	-47.6	10	Т

Table 4.7 Sagittarius Stream spectroscopic detections from Law et al. (2005) and Law & Majewski (2010). Leading and Trailing tails are denoted L and T respectively.

Attempts to model the dynamics of the tails via orbits met with contradictory results. Law et al. (2005) found that the trailing tail radial velocities are insensitive to the halo flattening q. The leading tail velocities, however, are best fit by a prolate halo with q = 1.25. This would have settled the matter, except that a prolate halo leads to the Sagittarius leading tidal tail precessing in the wrong direction, and thus not agreeing with stream detections on the sky.

This discrepancy remained until Law & Majewski (2010) formulated a consistent model of the Sagittarius leading and trailing tails in a triaxial halo. The halo, given analytically in Chapter 2, is logarithmic and contains three flattenings  $(q_1, q_2, q_z)$  and an angle  $\phi$  that rotates the halo in the X-Y plane. The second flattening  $q_2$  is set to unity, as it is the ratios of the flattenings that is physically significant. The best fit structure parameters found by Law & Majewski are  $(q_1, q_z, \phi) = (1.38, 1.36, 97^\circ)$ . They did not fit either the halo speed or scale length, holding these at  $v_{halo} = 115 \text{ km s}^{-1}$  and d = 12 kpc.

The best-fit triaxial structure parameters are troubling from the standpoint of Galactic formation. They represent a triaxial halo whose minor axis is in the Galactic plane. Stable orbits cannot form around the intermediate axis of a triaxial halo, and thus a disk cannot form. Law & Majewski acknowledge this fact, and suggest a possible interaction with another large scale piece of Galactic structure (such as the Magellanic clouds) as a remedy.

## 4.5 Galactic Rotation Curve

In the previous chapter, we created test rotation curves by perturbing the true rotation curve of a Galactic potential by normally distributed 20 km s<sup>-1</sup> errors. Figures 4.16 and 4.18 show the rotation curve data obtained by Xue et al. (2008). The Xue rotation curve data was not actually fit in the Orphan models. In the next section we present a simultaneous fit of three streams and the rotation curve. We will utilize rotation curve data from Simulation 1 of Xue et al. (2008, Table 3), which is reproduced in Table 4.8.

r (kpc)	$V_{cir} \ \mathrm{km} \ \mathrm{s}^{-1}$
7.5	$202 \pm 20$
12.5	$227\pm20$
17.5	$206\pm20$
22.5	$170 \pm 20$
27.5	$168 \pm 24$
32.5	$162 \pm 27$
37.5	$175 \pm 24$
42.5	$207\pm30$
47.5	$148 \pm 31$
55.0	$180 \pm 54$

Table 4.8 Galactic rotation curve from Xue et al. (2008)

## 4.6 Simultanous Triaxial Orbit Fit

With the streams discussed in the previous sections, and the machinery of Chapters 2 and 3, we now move to fitting the GD-1, Orphan, and Sgr stellar streams in a Galactic halo. Law et al. (2005) determined that Sgr cannot be adequately fit in an axisymmetric halo, we therefore use a more general triaxial form of the potential. We also make no attempt to fit the bifurcation of the leading tail because it is likely that this component of the structure arises from the internal structure of the dwarf galaxy, in particular its rotational properties (Peñarrubia et al. 2010 & Lokas et al. 2010). In Chapter 3 we showed that mock streams are unable to constrain parameters of a triaxial+spherical halo, we therefore do not attempt this either.

We construct a global fitness function containing the GD-1, Orphan, and Sagittarius streams, as well as the Galactic rotation curve (Equation 2.10). GD-1 and Orphan are fit in  $(\theta, \phi) = (l, b)$  coordinates, and Sagittarus is fit in  $(\theta, \phi) = (\Lambda_{\text{Sgr}}, B_{\text{Sgr}})$ . A particle swarm optimization was performed, and the search space included all physically valid parameter values. The number of particles in the optimization was varied from 200 to 1000. Particle swarm parameters  $w_i$ ,  $c_1$  and  $c_2$  were held constant at  $w_i = 0.9$ ,  $c_1 = c_2 = 0.3$ . The particle swarm results were searched for the best fit parameters, and allowed to run until the best fit solution remained unchanged for one day. The best fit parameters are given in Table 4.9. The fitness of this solution is  $\chi^2 = 6.28$ , and scale factors of  $S_{\text{Leading}} = 1.47$  and  $S_{\text{Trailing}} = 0.62$  were fit. The best fit and true orbits for GD-1 and Orphan are shown, along with the stream detections, in Figure 4.21. The simulated Sgr fit orbit with detections is shown in Figure 4.22. The rotation curve for this halo is shown in Figure 4.25.



Figure 4.21 Best fit orbits to GD-1 and Orphan streams in triaxial halo. We see good agreement in stream quantities. The best fit of the Orphan Stream distances is not as good as the axisymmetric  $v_{halo} = 73 \text{ km s}^{-1}$  halo in the previous section.

The parameters for the triaxial halo are perfectly consistent with Law & Majewski (2010). This is both encouraging and troubling. It is encouraging in the respect that the GD-1 and Orphan streams are consistent with a halo of this shape and orientation. It is troubling because, as Law & Majewski remarked, the halo orientation angle  $\phi$  places the intermediate axis of the halo in the plane of the Galaxy, which makes it difficult to understand how a stable Galactic disk formed.

Shown in Figure 4.24 are debris detections at 90 kpc from the Galactic center from Newberg et al. (2004). The simulated distances of the best fit Sgr trailing tail are not consistent with these detections. This result has two interpretations: either the debris associated with those detections actually does not belong to Sgr, or the triaxial halo model does not adequately represent the Milky Way halo. To test the



Figure 4.22 Best fit orbit to Sagittarius debris data in a triaxial halo. Overlayed is a 10,000 particle N-body simulation evolved for 4 Gyr along the best fit orbit. We see good agreement in velocities and distances. Fitting the distance scale factors provides a good first-order approximation to modeling the distances to the Sagittarius stream. Shown in black are the distances to tidal debris 90 kpc from the Galactic center, postulated by Newberg et al. (2004) to be Sagittarius debris.

first interpretation, we will ask the question: what would the halo speed need to be to have the simulated stream points intersect the detections, and preserve the total velocity of the particles? We must satisfy the energy relation:

$$E_{\text{total}}(R_{GC} = 65 \text{ kpc}) = E_{\text{total}}(R_{GC} = 90 \text{ kpc}),$$
 (4.1)

or more specifically

$$\frac{1}{2}mv^{2} - m\Phi_{\rm D}(R_{GC} = 65) - m\Phi_{\rm B}(R_{GC} = 65) + m\Phi_{\rm H}(R_{GC} = 65)$$
$$= \frac{1}{2}mv^{2} - m\Phi_{\rm D}(R_{GC} = 90) - m\Phi_{\rm B}(R_{GC} = 90) + m\Phi_{\rm H}(R_{GC} = 90)$$
(4.2)

Parameter	Value
$R_1$	$7.5 \pm 1.2 \; \mathrm{kpc}$
$v_{1,x}$	$-86\pm 6~{\rm km~s^{-1}}$
$v_{1,y}$	$-234 \pm 16 \text{ km s}^{-1}$
$v_{1,z}$	$-118 \pm 8 \text{ km s}^{-1}$
$R_2$	$29.2\pm4.4~\rm kpc$
$V_{2,x}$	$-183 \pm 26 \ \rm km \ s^{-1}$
$v_{2,y}$	$101 \pm 29 {\rm ~km~s^{-1}}$
$V_{2,z}$	$106 \pm 18 {\rm ~km~s^{-1}}$
$R_3$	$26.9\pm0.4~\rm kpc$
$v_{3,x}$	$244\pm4~{\rm km~s^{-1}}$
$v_{3,y}$	$-34\pm4~{\rm km~s^{-1}}$
$v_{3,z}$	$163\pm3~{\rm km~s^{-1}}$
$V_{halo,t}$	$120\pm3~{\rm km~s^{-1}}$
$q_1$	$1.44\pm0.07$
$q_z$	$1.41\pm0.05$
$\phi$	$94 \pm 1^{\circ}$
$\rm d_{halo,t}$	$18.2\pm1.3~\rm kpc$
$\chi^2$	6.28
$\begin{array}{c} {\rm V}_{1,{\rm z}} \\ {\rm R}_2 \\ {\rm V}_{2,{\rm x}} \\ {\rm V}_{2,{\rm y}} \\ {\rm V}_{2,{\rm z}} \\ {\rm R}_3 \\ {\rm V}_{3,{\rm x}} \\ {\rm V}_{3,{\rm x}} \\ {\rm V}_{3,{\rm y}} \\ {\rm V}_{3,{\rm z}} \\ {\rm V}_{{\rm halo},{\rm t}} \\ q_1 \\ q_z \\ \phi \\ {\rm d}_{{\rm halo},{\rm t}} \\ \chi^2 \end{array}$	$\begin{array}{c} 29.2 \pm 4.4 \ \mathrm{kpc} \\ -183 \pm 26 \ \mathrm{km \ s^{-1}} \\ 101 \pm 29 \ \mathrm{km \ s^{-1}} \\ 106 \pm 18 \ \mathrm{km \ s^{-1}} \\ 26.9 \pm 0.4 \ \mathrm{kpc} \\ 244 \pm 4 \ \mathrm{km \ s^{-1}} \\ -34 \pm 4 \ \mathrm{km \ s^{-1}} \\ 163 \pm 3 \ \mathrm{km \ s^{-1}} \\ 120 \pm 3 \ \mathrm{km \ s^{-1}} \\ 120 \pm 3 \ \mathrm{km \ s^{-1}} \\ 1.44 \pm 0.07 \\ 1.41 \pm 0.05 \\ 94 \pm 1^{\circ} \\ 18.2 \pm 1.3 \ \mathrm{kpc} \\ 6.28 \end{array}$

Table 4.9 Best particle swarm fit of kinematic and potential parameters of the GD-1, Orphan, and Sgr streams in the restricted triaxial case.

The detections at 90 kpc lie near the Galactic anticenter. We will therefore assume  $Y_{GC} = Z_{GC} = 0$ . Utilizing the Miyamoto-Nagai disk, bulge, and triaxial halo potentials from Chapter 2, we obtain:

$$-\frac{M_{\rm D}}{65} - \frac{M_{\rm B}}{65 + 0.7} + v_{\rm halo,t}^2 \ln(c_1 65^2 + d_{\rm halo,t}^2) = -\frac{M_{\rm D}}{90} - \frac{M_{\rm B}}{90 + 0.7} + v_{\rm halo,t,90}^2 \ln(c_1 90^2 + d_{\rm halo,t}^2)$$

Assuming constant halo scale length  $d_{\text{halo,t}} = 18.2$  kpc, and solving for the halo speed  $v_{\text{halo,t,90}}$ , we obtain  $v_{\text{halo,t,90}} = 115$  km s<sup>-1</sup>, which is not consistent with the fit halo speed to within one sigma confidence. We therefore conclude that the debris detections at 90 kiloparsecs from Newberg et al. (2006) are inconsistent with being members of the Sagittarius stream if the logarithmic halo model is a good description of the Milky Way.

This conclusion is limited to the scope of the logarithmic halo model. Every distance detection used to create this fit is within 50 kpc of the Sun, and they are all consistently fit with the logarithmic halo. The Milky Way halo may have a radial dependence that changes between 50 kpc and 90 kpc that can actually fit the debris at 90 kpc. We therefore stop short of concluding that the 90 kiloparsec detections are definitively excluded from Sgr stream membership; they are simply inconsistent with the logarithmic model. A topic of future work is investigating the radial dependence of the halo, and also improving upon the distance scale factor concept to allow distances at apogalacticon to be fit.

For thoroughness, we will conduct one more simultaneous fit, within a generalized triaxial halo with all Euler angles as parameters. A particle swarm optimization, conducted in a similar manner, gives the best fit parameters shown in Table 4.10. The fitness of this solution is  $\chi^2 = 6.03$ , and scale factors of  $S_{\text{Leading}} = 1.33$  and  $S_{\text{Trailing}} = 0.58$  were fit. The best fit and true orbits for GD-1 and Orphan are shown, along with the stream detections, in Figure 4.23. The simulated Sgr fit orbit with detections is shown in Figure 4.24. The rotation curve for this halo is shown in Figure 4.25.



Figure 4.23 Best fit orbits to GD-1 and Orphan streams in full triaxial halo. We see good agreement in stream quantities. The best fit of the Orphan Stream distances is not as good as the axisymmetric  $v_{halo} = 73 \text{ km s}^{-1}$  halo in the previous section.



Figure 4.24 Best fit orbit to Sagittarius debris data in full triaxial halo. Overlayed is a 10,000 particle N-body simulation evolved for 4 Gyr along the best fit orbit. We see better agreement in sky positions than the restricted triaxial model, and good agreement in velocities and distances. The Sagittarius dwarf is offset from its true position by  $10^{\circ}$  to show orbit consistency with dwarf radial velocity. Fitting the distance scale factors provides a good first-order approximation to modeling the distances to the Sagittarius stream. Shown in black are the distances to tidal debris 90 kpc from the Galactic center, postulated by Newberg et al. (2004) to be Sagittarius debris.

The conclusions for this fit are as follows:

- The angle φ is consistent with the results from Law & Majewski (2010). The angle θ is not consistent with zero while the angle ψ is consistent with zero. This result is more at odds with models of Galaxy formation than the Law & Majewski model, because now the Galactic disk is not aligned with any axis.
- The angle  $\phi$  is consistent with the stellar halo orientation found by Newberg & Yanny (2006). Our  $\theta$  is their  $\phi$ , our  $\phi$  is their  $\theta$  and our  $\psi$  is their  $\xi$ . In our naming convention, they find  $\phi \approx 70^{\circ}$ ,  $\theta \approx -5^{\circ}$  and  $\psi \approx 13^{\circ}$ . Our values for  $\theta$  and  $\psi$  disagree, but there is broad agreement for the value of  $\phi$  between this



Figure 4.25 Rotation curve fits for restricted and unrestricted triaxial halos. The noise at high r values is due both to the triaxiality of the halo as well as the numerical approximations used to calculate particle accelerations.

work, Newberg & Yanny, and Law & Majewski. This is compelling, because Newberg & Yanny fit the triaxial orientation of stars in the Galaxy, without referring to any specific form of the potential. It is not necessary for the stars to have the same orientation as the dark matter. Our result of a consistent  $\phi$ value is important because it suggests a possible link between the dark matter halo and the stellar population of the Galaxy in this dimension.

- As before, the simulated trailing stream distances do not coincide with the Sgr detections from Newberg et al. (2004). This does not necessarily rule out stream membership of those detections, but rather simply states that they are not consistent with a model evolved in a logarithmic halo.
- This simultaneous fit has sacrificed the Orphan Stream distances to obtain better fits elsewhere. The low mass, axisymmetric halo with  $v_{halo} = 73 \text{ km s}^{-1}$ remains the best fit to the Orphan distances.

Parameter	Value
$R_1$	$7.6\pm1.2~\rm kpc$
$v_{1,x}$	$-85\pm7~{ m km~s^{-1}}$
$v_{1,y}$	$-232 \pm 18 \text{ km s}^{-1}$
$v_{1,z}$	$-117 \pm 9 {\rm ~km~s^{-1}}$
$R_2$	$26.5\pm4.6\;\mathrm{kpc}$
$v_{2,x}$	$-180 \pm 28 \; \rm km \; s^{-1}$
$v_{2,y}$	$96 \pm 32 \text{ km s}^{-1}$
$v_{2,z}$	$108 \pm 17 \ {\rm km \ s^{-1}}$
$R_3$	$23.7\pm1.1~\rm kpc$
$v_{3,x}$	$243 \pm 19 \text{ km s}^{-1}$
$v_{3,y}$	$-32 \pm 28 \text{ km s}^{-1}$
$v_{3,z}$	$174\pm9~{\rm km~s^{-1}}$
$v_{halo,t}$	$126\pm9~{\rm km~s^{-1}}$
$q_1$	$1.33\pm0.16$
$q_z$	$1.52\pm0.14$
heta	$-50^{\circ} \pm 18^{\circ}$
$\phi$	$86^{\circ} \pm 11^{\circ}$
$\psi$	$1^{\circ} \pm 6^{\circ}$
$d_{\rm halo,t}$	$22.2\pm3.3~\rm kpc$
$\chi^2$	6.03

Table 4.10 Best particle swarm fit of kinematic and potential parameters of the GD-1, Orphan, and Sgr streams in the unrestricted triaxial halo.

• In the same manner as Law & Majewski, these results do not reproduce the bifurcation in the Sagittarius leading tidal tail. Recent studies (Peñarrubia et al., Lokas, et al.) indicate the bifurcation may be a result of internal Sagittarius dynamics.

# 4.7 Discussion

This chapter has accomplished two primary goals. The first is a thorough analysis of three Galactic tidal streams: the GD-1 Stream, the Cetus Polar Stream, and the Orphan Stream. These streams, individually, provide powerful constraints of the kinematics of the compact progenitors, and the Orphan Stream helps to constrain the total halo mass. Second, GD-1 and Orphan have been combined with the Sagittarius Stream to provide constraints on a fully triaxial dark matter halo. In the restricted case, results consistent with previous authors have been recovered. In the unrestricted case, new halo orientation angles have been found.

These constraints are not without trade-offs though: the kinematic constraints have been sacrificed. While the fully triaxial halo is capable of satisfying all Sagittarius constraints, the low mass, axisymmetric halo remains the best fit to the Orphan Stream distances. Additionally, the new halo orientation angles remain at odds with our understanding of Galaxy formation. Specifically, a Galactic disk cannot form in a halo whose minor axis lies in the plane of the disk. One possible explanation for this discrepancy is that the Sagittarius Dwarf may have interacted with a similarlysized body in the past, which perturbed its path enough to affect the structure of its tidal tails. Chapter 5 will further discuss future work in this field.

# CHAPTER 5 CONCLUSIONS AND FUTURE WORK

This work has accomplished two primary goals. The first is the development of a general orbit fitting method that has been tested for robustness using simulated tidal streams. The second is the elaboration of four Galactic tidal streams: the Stream of Grillmair & Dionatos (GD-1), the Cetus Polar Stream, the Orphan Stream, and the Sagittarius Dwarf Tidal Stream. While none of these streams is understood to the fullest possible extent, an orbit model has been developed that incorporates them all, in the latest triaxial Galactic halo potential, with the newest measurements of the Galactic rotation curve. The results presented herein are consistent with previous findings, and present challenges to the most current models of galactic formation.

Orbit fits were conducted for cases including single, double, and triple tidal streams within a variety of Galactic potentials. The orbit fitting method of Chapter 2 was shown to be robust in cases where rotation curves were used, and velocities of tidal stream stars cooresponded to orbit values. Out on the far ends of streams, where the stream stars can deviate from the orbit, large velocity errors are required to recover kinematic and potential parameters. Using simulations designed to mimic real streams in the Milky Way halo, it was found that:

- Stream kinematics for simulated streams can be constrained, with or without rotation curve fitting.
- Adding measurements of halo rotation curves when fitting tidal debris streams aids in finding the best fit halo parameters.
- The total mass of a spherical halo is well constrained, even when the halo has a different model than that used to create the stream.
- Fits of the flattening q suffer systematic biases. We have shown these biases to be caused by the dispersion in stream quantities, and if the stream mirrored its orbit perfectly, the halo parameters would be recovered.

- At the ends of streams, our clipped mean technique determines stream velocities and errors that are inconsistent with the orbit used to create the stream. This causes the optimization algorithm to converge to poor minima. Expanding radial velocity and angle errors to experimentally appropriate levels alleviates this difficulty, and leads to good fits.
- Gradient search is an effective method for finding best fit kinematics of a single stream in a known potential, but quickly collapses when more streams and parameters are introduced. We show that particle swarm optimization is a better choice for multiple streams.
- Two streams chosen to mimic the GD-1 and Orphan Streams in a triaxial halo are able to constrain the structure parameters (mass and flattenings), but are unable to recover the scale length. The constraints on the scale length are obtained by introducing a Sgr emulator stream.
- We are unable to recover parameters from a triaxial+spherical halo with three streams chosen to emulate GD-1, Orphan, and Sagittarius.
- Given a true halo that is triaxial+spherical, stream parameters are not recovered when fit with a triaxial model. This provides two a crucial insights: we are able to determine that a triaxial halo is not a good fit to streams generated in a triaxial+spherical halo, and conversely, if true streams are not fit well in a triaxial halo, an additional halo component may be responsible for the discrepancy.

With the robustness of the orbit fitting method established, we moved on to fitting the GD-1, Cetus Polar, and Orphan Streams individually within the logarithmic dark matter halo model. The GD-1 stream was shown to be on a highly retrograde orbit with an eccentricity  $e = 0.33 \pm 0.02$  (one sigma error) and an inclination to the Galactic plane of  $i \sim 35 \pm 5^{\circ}$ . Best fit kinematics at  $(l, b) = (172.3^{\circ}, 57.2^{\circ})$  are  $(R, v_x, v_y, v_z) = (8.4 \pm 0.8 \text{ kpc}, -89 \pm 2 \text{ km s}^{-1}, -236 \pm 6 \text{ km s}^{-1}, -115 \pm 3 \text{ km s}^{-1})$ . No constraints were placed on the parameters of the logarithmic halo using the GD-1 Stream. The Cetus Polar Stream was discussed and fit, but the sparse detections also precluded any logarithmic halo parameter constraints. The best fit kinematics for CPS at  $(l, b) = (144^{\circ}, -71^{\circ})$  are  $(R, v_x, v_y v_z) = (31.1 \text{ kpc}, -103 \text{ km s}^{-1}, 80 \text{ km s}^{-1}, 76 \text{ km s}^{-1})$ .

Fits to the Orphan Stream resulted in an orbit with an eccentricity e = 0.7and inclination  $i \sim 34^{\circ}$ . The Orphan Stream distances were found to be best fit by a low mass, axisymmetric logarithmic halo with  $v_{halo} = 73$  km s<sup>-1</sup> and kinematics at  $(l, b) = (218^{\circ}, 53.5^{\circ}, 30 \text{ kpc})$  of  $(\text{R}, \text{v}_{\text{x}}, \text{v}_{\text{y}}, \text{v}_{\text{z}}) = (-156 \pm 10 \text{ km s}^{-1}, 79 \pm$  $1 \text{ km s}^{-1}, 107 \pm 9 \text{ km s}^{-1})$ . Of the models that we tried, the three low-mass exponential disk models (1-3) fit either the Orphan Stream data or the Xue et al. (2008) rotation curve, but not both. The two low halo mass models (models 5,7) with a higher mass M-N disk fit the best, with a logarithmic halo fitting about the same as an NFW profile. The two higher halo mass models (models 4,6) are poorer fits to both the Orphan Stream and the Xue et al. (2008) rotation curves. The distance to the far ends of the Orphan Stream is a powerful probe into the total halo mass. Additionally, it was found that a dwarf galaxy with mass  $M \sim 2 \times 10^6 M_{Sun}$  and scale radius  $r_s = 0.2$  kpc, evolved on the best fit orbit for 3.945 Gyr was able to broadly recover the density profile of F-turnoff stars in the Orphan Stream.

Combining the GD-1 and Orphan streams with previously obtained Sagittarius observations allowed a simultaneous model to be fit within a logarithmic triaxial dark matter halo. When two Euler angles are restricted ( $\theta$  and  $\psi$ ), we recover the result of Law & Majewski (2010). Fitting over all orientation angles finds angles  $\theta = -50^{\circ}$ ,  $\phi = 89^{\circ}$  and  $\psi$  consistent with zero. While this is a novel result, it does not resolve difficulties with the Law & Majewski result regarding the orientation of the halo's minor axis. Nor does this model attempt to reproduce the bifurcation of the Sagittarius leading tail. The detections at 90 kpc from Newberg et al. (2007) are not consistent with the best fit model of Sagittarius within a logarithmic triaxial halo model. This does not completely discount their stream membership, but instead makes the statement that if they are Sagittarius debris, the logarithmic halo is insufficient to describe the dynamics of the Sagittarius stream.

## 5.1 Future Work

This Thesis opens a door to a wide variety of future work, only some of which can be described here. Work being done to fit the Orphan stream stellar density will be described, followed by a general discussion of future research directions.

### 5.1.1 Fitting The Orphan Stream Stellar Density

The stellar density of the Orphan stream was shown in the previous chapter in Figures 4.13 and 4.14. Photometrically selected F-turnoff stars show a stellar overdensity near ( $\Lambda_{\text{Orphan}}, B_{\text{Orphan}}$ ) = (22°, 0°), which is presumed to be the core of the Orphan stream projenitor. Figure 4.14 shows the F-turnoff stellar density as a function of  $\Lambda_{\text{Orphan}}$  along the Orphan coordinate system equator. The overall global behavior, shown in Figure 4.20, was mimicked using a Plummer model with  $M_{\text{total}} = 10^6 \text{ M}_{\text{Sun}}$  and  $r_s = 0.2 \text{ kpc}$  evolved for 3.945 Gyr.

While these parameters provide an interesting baseline, a research question emerges: can we use methods similar to orbit fitting to fit the true Orphan stream stellar density? We develop a fitness function that allows us to evaluate how well a model simulation fits the data. Next, a method for performing massively distributed N-body simulations will be described and tested.

### 5.1.1.1 Construction of a Fitness Function

The Orphan stream stellar density lends itself straightforwardly to the construction of a fitness function. We will use the best fit Orphan Stream orbit derived in the previous chapter. We evolve that orbit back for a time  $t_{\text{back}}$  and place N mass points representing a Plummer sphere of mass M and scale length  $r_s$  at the predicted orbit position and velocity. The Plummer sphere is evolved forward for a time t to produce a disrupted stream. Two times are used because of energy segregation: the enclosed mass of the dwarf galaxy decreases as it is tidally disrupted. As such, it speeds up along its orbit and will arrive at a destination point faster than the orbit will predict.

At this point, the positions of the stream mass points are converted to Orphan Stream coordinates and binned in  $\Lambda_{\text{Orphan}}$  between  $\Lambda_{\text{Orphan}} = -50^{\circ}$  and  $\Lambda_{\text{Orphan}} =$   $+50^{\circ}$  to produce a simulated stream density histogram similar to Figure 4.20.

The number of simulation particles requires special consideration. A sufficient number of particles are needed to ensure that the resulting histogram is an adequate representation of the true stream. We therefore need to avoid the small N limit. In addition, the number of particles affects the N-body simulation time. A large enough particle count is required to get a good distribution, but a small enough count is required to avoid excessively long computation times. We therefore choose a particle count between ten and one hundred thousand. On an averge desktop computer, a typical Orphan N-body with this number of particles takes on the order of hours. As will be discussed later, ensuring a sufficiently long computation time is also an important consideration.

As long as enough particles are present to represent the stream density distribution, the results of the simulations should not depend explicitly on the number of particles. Since we are attempting to match the true Orphan stream density, only the ratios between the bins are physically relevant. It would therefore seem that we should divide each bin by the total number of particles in the simulation. This presents a troubling obstacle: what is the appropriate normalization factor for comparison with the real density data? We do not know the total number of stars in the entire Orphan stream. Nor do we know the ratios between the selected F turnoff stars and other populations in the dwarf. We only know the number of F turnoff stars that are in the range  $\Lambda_{\text{Orphan}} = -50^{\circ}$  to  $\Lambda_{\text{Orphan}} = +50^{\circ}$ , and we are explicitly assuming that the density data by the total number of F turnoff stars between  $\Lambda_{\text{Orphan}} = -50^{\circ}$  and  $\Lambda_{\text{Orphan}} = +50^{\circ}$ . We also divide the simulation density by the number of simulation particles between  $\Lambda_{\text{Orphan}} = -50^{\circ}$  and  $\Lambda_{\text{Orphan}} = +50^{\circ}$ . This ensures a consistent normalization between both.

With the normalized data and simulation histograms established, we construct a fitness function by summing the residuals between the data and simulation histograms

$$\chi^2 = \sum_{i} \left( \frac{N_{norm, model, i} - N_{norm, data, i}}{\sigma_{N, data, i}} \right)^2, \tag{5.1}$$

where  $\sigma_{N,data,i} = \frac{\sqrt{N_{data,i}}}{N_{data,i}}$ ,  $N_{norm,model,i} = \frac{N_{model,i}}{N_{model}}$  and  $N_{norm,data,i} = \frac{N_{data,i}}{N_{data}}$ .  $N_{data}$  ( $N_{model}$ ) is the number of stars (particles) in the range  $\Lambda_{\text{Orphan}} = -50^{\circ}$  to  $\Lambda_{\text{Orphan}} = +50^{\circ}$  in the data (simulation).

### 5.1.1.2 Running the N-body Simulations: Volunteer Computing

We fix the Orphan Stream orbit parameters and Galactic potential to the best fit values from the previous chapter. Here, we fit only the set of dwarf parameters  $\vec{Q} = (M, r_s, t_{back}, t).$ 

Minimizing a fitness function with a particular search method is a solved problem. In the previous chapters, we have been able to minimize the orbit metrics straightforwardly because orbit evaluations take on the order of seconds to complete. Particle swarm and gradient search optimizations for orbit fitting can be conquered by a desktop computer in a day. For this problem, however, a single fitness evaluation requires an N-body simulation, which have computation times on the order of hours. A single desktop computer, even with a supremely optimized search method, is incapable of approaching this problem.

To remedy this difficulty, we have modified a Barnes & Hut (1986) treecode to operate over the Berkeley Open Infrastructure for Network Computing (BOINC) framework used by the Milkyway@Home project. This framework allows volunteer computers to perform one fitness evaluation each, and have results returned to the central Milkyway@Home server, which then optimizes the dwarf parameters using the Framework for Generic Disributed Optimization (FGDO, Desell [2009]). The orbital and halo parameters are kept constant, and are given by Table 4.5. What is allowed to vary are the relevant Orphan progenitor parameters: mass, scale length, and backward & forward evolution times.

### 5.1.1.3 Verifying Dwarf Parameters with Test Density Fits

In a manner similar to Chapter 3, we first test our fitness function and search methods using test density distributions. We construct a test Orphan stream density distribution using the best fit  $v_{halo} = 73$  km s<sup>-1</sup> orbit model from the previous chapter. We choose M = 16  $M_u \approx 3.6 \times 10^6$  M<sub>Sun</sub>,  $r_s = 0.2$  kpc,  $t_{\text{back}} = 4$  Gyr, t = 3.945 Gyr, and  $N = 10^5$ .

We fit the simulated Orphan Stream stellar density using the FGDO framework of Desell (2009). The N-body model is composed of a 10,000 particle Plummer sphere with mass, scale radius, and evolution time ranges given in Table 5.1. In a manner similar to Desell & Willett, et al. (2010), we utilize a differential evolution search with population size of 300, best parent selection, and a pair weight of 0.5(Mezura-Montes et al., 2006). Workunits consisting of a set of dwarf parameters were evaluated by the Milkyway@home users, and returned to the server for analysis and optimization. Validation was employed to ensure consistent fitness evaluations for the same sets of parameters. For each generation of the differential evolution search, the best, median, and average fitness values are recorded and measured over time. Figure 5.1 shows the evolution of the first Orphan Stream model. The optimizations were allowed to run until no apparent change existed in the best fit solution for a period of 24 hours. The best fit parameters to the simulated dataset are M = 17.6  $M_u$   $\approx$  3.951  $\times$   $10^6~{\rm M_{Sun}},~r_s$  = 0.22 kpc,  $t_{\rm back}$  = 3.97 Gyr, t = 3.91 Gyr. The simulated stream histogram is shown in red in Figure 5.2 and the best fit histogram is shown in green. The best fit solution to the test density histogram successfully reproduces the parameters of the simulated dwarf galaxy. Work is currently underway to extend this infrastructure to fit the actual Orphan Stream stellar density given in Chapter 4.

Parameter	Lower Bound	Upper Bound
Plummer Mass	$1.0 M_u$	$50 M_u$
Plummer scale radius	$0.05 \ \mathrm{kpc}$	$1 \ \rm kpc$
Orbit evolution time	1 Gyr	$5 { m Gyr}$
N-body evolution time	1 Gyr	$5 { m Gyr}$

Table 5.1 Simulated dwarf galaxy parameter ranges.

## 5.1.1.4 Implications of Dark Matter in the Dwarf

The results given above are a step forward in N-body modeling of streams. However, there is a fundamental assumption being made: all of the simulation particles are being compared to the F-turnoff stellar distribution of the Orphan Stream. Put another way, we are creating an N-body simulation of just F-turnoff



Figure 5.1 Progress of the best, average, and median validated individuals for an asynchronous differential evolution search with a population of 300 individuals. The searches were run until the best fit solution remained unchanged for a period of 24 hours.

stars, and neglecting all other stellar types, as well as dark matter. A more complete model of the Orphan progenitor should include an initial mass function to account for other stellar types, as well as dark matter.

Previous authors (e.g. Law et al 2005) model dwarf galaxies as a single set of N-body particles that represent both light and dark matter. The rationale of this decision is that the light and dark matter both respond the same to the Galactic potential. This approach is not appropriate for the Orphan Stream problem, because we are attempting to constrain the total mass of the Orphan progenitor using observations of only the stars. If we neglect dark matter completely, its contribution to the mass is lost.

Introducing dark matter into the problem requires careful thought and consideration. Within dwarf galaxies, stars are typically found inside larger dark matter


Figure 5.2 Simulated and best fit Orphan stream density histogram as a function of Orphan longitude  $\Lambda_{\text{Orphan}}$ . The simulated histogram is generated with parameters  $M = 16 \ M_u \approx 3.3 \times 10^6 \ M_{\text{Sun}}, r_s = 0.2 \ \text{kpc}, t_{\text{back}} = 4 \ \text{Gyr}, t = 3.945 \ \text{Gyr},$  and  $N = 10^5$ . The best fit parameters are  $M = 17.6 \ M_u \approx 3.951 \times 10^6 \ M_{\text{Sun}},$   $r_s = 0.22 \ \text{kpc}, t_{\text{back}} = 3.97 \ \text{Gyr}, t = 3.91 \ \text{Gyr}.$ 

halos. We create toy model Plummer spheres representing the two populations: stars (mass =  $1.1 \times 10^6 M_{Sun}$ ,  $r_s = 0.2 \text{ kpc}$ ) and dark matter (mass =  $1.3 \times 10^6 M_{Sun}$ ,  $r_s = 0.5 \text{ kpc}$ ). These numbers were chosen such that the stellar and dark matter distributions have approximately the same density. The true mass ratio between these components is a topic for further study. The smaller stellar Plummer sphere is placed inside the larger dark matter sphere, and allowed to evolve along the best-fit Orphan Stream orbit. The density profile of this evolution is shown in Figure 5.3. We see that, as we expected, the dark matter is tidally stripped first, and an overdensity of dark matter can be clearly seen near  $\Lambda_{\text{Orphan}} = -40^{\circ}$ . This implies the mass to light ratio of a two-component dwarf galaxy model is expected to vary along the stream.

The observation that the light and dark matter do not have the same density



Figure 5.3 Simulated 2-component Orphan stream density histogram as a function of Orphan longitude  $\Lambda_{\text{Orphan}}$ . The stellar simulated histogram is generated with parameters  $M = 5 M_u \approx 1.1 \times 10^6 \text{ M}_{\text{Sun}}$ ,  $r_s = 0.2 \text{ kpc}$ ,  $t_{\text{back}} = 4 \text{ Gyr}$ , t = 3.945 Gyr, and  $N = 10^5$ . The dark matter simulated histogram is generated with parameters  $M = 6 M_u \approx 1.3 \times 10^6 \text{ M}_{\text{Sun}}$ ,  $r_s = 0.5 \text{ kpc}$ ,  $t_{\text{back}} = 4 \text{ Gyr}$ , t = 3.945 Gyr, and  $N = 10^5$ . The overdensity of dark matter near  $\Lambda_{\text{Orphan}}$  is caused by the outer dark matter population being tidally disrupted before the stars.

profile raises an important question about the results we obtained in the previous section. Do the properties we fit apply only to the stellar matter in the Orphan Stream, or is there a coupling between the light and dark matter which makes the parameters we fit some combination of both populations? Also, what do the times represent? Are they the true evolution times of the entire progenitor, or the times since the light matter became "unshielded" by the dark matter? These are all questions for future work.

## 5.2 Research Directions

While the simultaneous model presented in this work is the best fit to the analyzed tidal streams, it leaves open several questions about the Galaxy itself. The Galaxy has been modeled as a static, time independent gravitational potential mainly for the reason of simplicity. While a variety of shapes have been examined, in no situation was the potential allowed to be dynamic. The fundamental assumption was that for at least the last 4 Gyr, a static Galactic potential would be broadly appropriate. A more dynamic model of the Galaxy, one which includes a time varying halo mass to incorporate accretion of the very tidal debris that we have examined, is the next logical step. This introduces a variety of new degrees of freedom. What is the form of the time dependence? What are its parameters? How can we begin to analyze it, and does the data exist to place any constraints on it?

Not only can the Galaxy be time varying, but the progenitors of the streams can also be. The counterintuitive results obtained for the triaxial orientation angle by Law & Majewski (2010), which have been corroborated by this work, present drastic problems to the current framework of Galactic formation. A triaxial halo with the minor axis aligned in the plane of the disk is inconsistent with current understanding of disk formation. Why have model fits to Sgr led us to this result? Has there been some interaction with Sgr in the past that could re-orient its tidal tails? Or is the halo orientation time dependent, and what we're seeing is a local, unstable effect?

Perhaps more fundamentally, however, is the concept that the Milky Way's dark matter distribution is not smooth. Instead of being describable as a smooth potential, it is more likely to be a collection of subhalos, which have broad density fluctuations. The computational capability exists to model such distributions, but as was mentioned before: does enough data exist to constrain any parameters?

While the Galaxy presents its own opportunities for future work, so do the very streams we have attempted to understand here. The GD-1 stream contains density fluctuations that cannot be understood as multiple perigalactic passages (Grillmair, private conversation). The Palomar 5 tidal stream has been mapped in the SDSS and contains density fluctuations that are not understood. More mapping of the Orphan Stream off of the SDSS footprint is needed, to test the hypothesis of the progenitor lying along the edge of the data. As was shown earlier, mapping the Orphan Stream distances of the far end of the tail will provide a powerful probe of the total halo mass. The bifurcation of the Sagittarius leading tidal tail is still a matter of contention. Recent work (Peñarrubia et al., 2010; Lokas et al., 2010) has shown it is a possible effect of internal dwarf rotation, but the matter is far from settled. The Cetus Polar Stream has only been mapped in segments, and a complete picture of the south Galactic cap is needed to even begin to understand its density profile. Only by mapping both stream kinematics and stellar density, within an elaborate model of the Galaxy, will a more complete understanding of Milky Way dynamics be obtained.

It is clear that the field of Galactic tidal streams has many challenges awaiting it. This Thesis lays some ground work to understanding these challenges, but the brightest days of understanding Milky Way substructure are ahead.

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## APPENDIX A GD-1 Stream Spectroscopic Candidates

SpecID	Ν	$\alpha$	δ	$v_{\rm gsr}$	$\sigma(v)$	RV	$g_0$	$(u - g)_0$	$(g - r)_0$	[Fe/H]	$\sigma([{\rm Fe}/{\rm H}])$	log g	$\sigma(\log g)$	$\mu_l$	$\mu_b$
		0	0	${\rm km~s^{-1}}$	${\rm km}~{\rm s}^{-1}$	$\rm km~s^{-1}$	mag	mag	mag	dex	dex	dex	dex	mas/yr	mas/yr
1154-53083-266	1	125.683891	-0.439561	113.4	7.4	264.7	19.107	0.909	0.275	-1.89	0.08	3.46	0.25	3.5	-4.3
1154 - 53083 - 145	1	126.577150	-0.439386	97.8	7.8	249.4	19.102	0.904	0.180	-2.14	0.07	3.62	0.23	12.9	-10.5
1154 - 53083 - 155	1	126.645407	-0.340477	109.0	3.6	260.3	18.117	0.935	0.275	-2.06	0.04	3.30	0.22	11.6	-3.9
1760-53086-339	2	131.406984	9.770413	65.8	14.5	186.9	18.389	0.909	0.227	-2.18	0.11	4.05	0.29	9.4	-8.4
2671 - 54141 - 432	2	132.157853	11.135851	77.3	6.0	193.9	18.780	0.841	0.406	-2.24	0.04	3.54	0.22	10.0	-5.2
2667 - 54142 - 427	2	132.367656	11.325495	70.5	4.0	186.5	17.433	0.987	0.420	-2.17	0.02	2.77	0.24	6.7	-7.4
2667 - 54142 - 604	2	133.073392	12.266884	62.3	2.5	175.1	15.859	1.159	0.486	-2.08	0.03	2.22	0.15	8.4	-6.1
2319-53763-347	3	138.399875	22.413022	38.1	11.6	114.5	19.451	0.843	0.243	-2.24	0.22	3.80	0.09	13.2	-7.7
2319-53763-358	3	138.450751	22.434572	40.4	4.7	116.7	17.996	0.865	0.259	-1.93	0.02	3.90	0.10	12.5	-9.7
2319-53763-387	3	138.669378	22.559736	48.9	11.8	124.7	19.603	0.874	0.385	-2.07	0.07	3.68	0.30	12.1	-7.9
2889 - 54530 - 311	4	142.787815	29.461085	-0.9	4.2	47.8	17.454	0.972	0.417	-1.98	0.04	3.23	0.28	10.7	-9.4
2914 - 54533 - 297	4	142.978804	29.136307	-3.0	3.9	46.9	18.139	0.925	0.189	-2.09	0.04	3.94	0.16	8.8	-9.3
2889-54530-240	4	143.339524	29.620398	-6.2	4.7	41.7	17.933	0.883	0.256	-2.14	0.03	3.63	0.17	9.6	-6.7
2889 - 54530 - 204	4	143.424098	29.036453	-3.6	5.1	46.5	17.822	0.872	0.271	-2.21	0.05	3.28	0.20	6.7	-4.5
2889-54530-215	4	143.453188	29.120632	-3.4	1.7	46.4	14.981	1.194	-0.237	-2.09	0.11	3.20	0.17	9.5	-10.9
2889 - 54530 - 238	4	143.552948	29.477508	-4.3	4.6	44.1	17.843	0.888	0.293	-2.21	0.01	3.60	0.18	8.4	-9.1
2889-54530-225	4	143.597836	29.802697	-6.1	1.7	41.0	14.845	1.196	-0.102	-1.98	0.06	3.34	0.09	13.7	-11.1
2914 - 54533 - 171	4	143.718962	29.874100	-3.6	8.0	43.2	19.432	0.747	0.285	-2.26	0.08	3.33	0.56	15.4	-8.7
2914 - 54533 - 458	4	144.107345	30.737924	-9.8	4.8	33.6	18.716	0.779	0.216	-2.26	0.05	4.41	0.10	13.1	-6.5
2914-54533-453	4	144.120941	30.947724	-1.4	4.4	41.2	17.955	0.827	0.220	-2.10	0.02	3.79	0.28	14.3	-6.3
2914-54533-509	4	144.275720	30.280468	-4.4	3.9	40.6	18.292	0.909	0.187	-2.25	0.01	3.46	0.31	9.5	-12.2
2914 - 54533 - 515	4	144.487234	30.410401	-4.7	3.6	39.8	18.247	0.855	0.232	-1.82	0.04	3.72	0.15	10.5	-1.6
2914 - 54533 - 540	4	144.561728	30.968385	-1.9	11.0	40.4	20.190	0.854	0.344	-1.95	0.10	3.46	0.21	12.6	-3.8
2567 - 54179 - 396	5	157.181542	44.514349	-67.9	6.8	-85.0	18.745	0.841	0.254	-2.08	0.04	3.78	0.19	13.9	1.8
2567 - 54179 - 246	5	157.565788	43.643316	-73.0	6.0	-87.1	18.630	0.934	0.167	-2.23	0.01	3.71	0.18	12.9	-0.9
2567 - 54179 - 238	5	157.691771	43.797748	-71.9	12.7	-86.7	19.546	0.908	0.206	-2.21	0.02	3.27	0.16	10.1	-5.6
2567 - 54179 - 189	5	157.817344	43.923322	-70.1	11.9	-85.5	19.761	0.772	0.342	-1.88	0.07	4.23	0.36	9.0	3.5
2567 - 54179 - 211	5	157.889540	43.500286	-65.2	10.0	-79.0	19.309	0.901	0.205	-1.89	0.15	3.74	0.07	9.2	-0.3
2567 - 54179 - 212	5	158.051293	43.644022	-67.6	5.2	-82.1	18.349	0.839	0.208	-1.90	0.07	3.67	0.26	9.7	-0.2
2567 - 54179 - 170	5	158.137638	44.086320	-58.2	9.8	-74.4	19.112	0.863	0.201	-2.04	0.06	3.64	0.17	14.4	-1.7
2567-54179-491	5	158.437074	44.466389	-69.2	9.1	-87.1	18.242	0.880	0.209	-2.02	0.09	3.95	0.16	8.9	-1.9

SpecID	Ν	α	δ	$v_{\rm gsr}$	$\sigma(v)$	RV	$g_0$	$(u-g)_0$	$(g - r)_0$	[Fe/H]	$\sigma({\rm [Fe/H]})$	log g	$\sigma(\log g)$	$\mu_l$	$\mu_b$
		0	0	${\rm km~s}^{-1}$	${\rm km}~{\rm s}^{-1}$	${\rm km~s^{-1}}$	mag	mag	mag	dex	dex	dex	dex	mas/yr	mas/yr
2567-54179-511	5	158.700621	44.530392	-68.9	4.4	-87.2	18.083	0.954	0.237	-2.22	0.01	3.18	0.25	12.1	1.3
2557 - 54178 - 498	5	158.747249	44.686776	-74.2	3.9	-93.2	16.794	1.048	0.476	-2.12	0.02	2.60	0.27	13.5	2.7
2410-54087-297	6	162.371533	47.207633	-87.0	6.9	-118.3	18.893	0.885	0.255	-2.26	0.02	3.46	0.20	15.4	-1.3
2410-54087-236	6	162.451620	48.002695	-85.2	4.2	-119.5	18.305	0.896	0.250	-1.71	0.04	3.51	0.21	8.6	4.8
2410-54087-288	6	162.547222	46.990942	-88.0	12.2	-118.7	19.344	0.893	0.164	-2.08	0.08	3.78	0.14	9.6	-3.8
2390-54094-256	6	162.557763	47.251097	-95.0	2.6	-126.6	16.945	1.048	0.036	-1.83	0.00	3.89	0.19	16.1	-1.5
2410-54087-380	6	162.743760	48.903984	-85.3	5.1	-123.1	18.632	0.840	0.269	-1.80	0.07	3.81	0.05	12.7	-1.6
2410-54087-173	6	163.799206	47.813588	-88.8	7.1	-123.6	18.828	0.845	0.200	-2.06	0.07	3.86	0.30	8.9	-1.8
2410-54087-539	6	164.064054	49.034036	-92.2	8.2	-131.6	19.293	0.925	0.169	-1.68	0.11	3.10	0.30	8.1	-1.7
2410-54087-501	6	164.334475	48.472850	-81.1	7.8	-118.8	19.401	0.925	0.202	-1.76	0.08	3.80	0.28	7.3	0.0
2410-54087-504	6	164.371325	48.224481	-90.6	8.8	-127.4	19.409	0.841	0.208	-1.75	0.07	4.30	0.14	11.3	-0.6
2390-54094-565	6	164.616981	49.202693	-81.1	2.0	-121.6	16.919	1.105	0.487	-1.86	0.01	2.93	0.19	6.9	3.7
2390-54094-615	6	165.199453	48.669262	-92.4	3.9	-131.5	17.895	0.896	0.385	-2.24	0.02	3.81	0.16	11.8	-1.7
2410-54087-637	6	165.381730	48.671045	-87.1	6.2	-126.4	18.928	0.887	0.206	-2.29	0.05	3.84	0.12	7.8	-0.2
2539 - 53918 - 406	7	217.430060	59.067783	-163.8	3.0	-297.5	15.805	1.226	-0.133	-2.11	0.03	3.29	0.12	5.9	3.7
2539 - 53918 - 070	7	219.681591	57.897316	-148.9	10.7	-283.7	17.970	0.862	0.199	-2.15	0.02	3.73	0.04	2.5	10.8
2539 - 53918 - 596	7	219.898491	58.258368	-157.6	5.5	-293.3	17.035	1.197	-0.167	-2.10	0.26	2.92	0.43	1.0	2.9